V. B. S. Purvanchal University, Jaunpur

Syllabus

M.A./M.Sc.Mathematics

M.A./M.Sc.(Mathematics) Previous

Sr.	Name of the Papers	Theoretical/Practical/ Viva-voce/Assignment	Maximum Marks	Duration (hours)
1.	Abstract Algebra	Theoretical	80	3.00
2.	Real Analysis	Theoretical	80	3.00
3	Topology	Theoretical	80	3.00
4	Complex Analysis	Theoretical	80	3.00
5.	Differential Geometry	Theoretical	80	3.00
6.	**Optional** **Group-A** Differential Equation or **Group-B** Advanced Discrete Mathematics or **Group-C** Spherical Astronomy or **Group-D** Special theory of relativity or **Group-E** Integral Equation and boundary value problem.	Theoretical	100	3.00
7.	Viva-Voce	Viva-Voce	100	
Total Marks = 600				

M.A./M.Sc.(Mathematics) Final

Sr.	Name of the Papers	Theoretical/Practical/ Viva-voce/Assignment	Maximum Marks	Duration (hours)
1.	Measure and Integration	Theoretical	75	3.00
2.	Partial Differential Equation	Theoretical	75	3.00
3	Mechanics	Theoretical	75	3.00
4	Functional Analysis		75	3.00
5	**Two Optional Papers** Two papers out of the following have to be chosen keeping in view the prerequisite and suitability of the combination. 1. Fluid Mechanics 2. Difference Equation 3. Mathematics of Finance 4. Information Theory 5. Algebraic Topology	Theoretical	100	3.00
6	6. Operations Research 7. General relativity and Cosmology 8. Differential Geometry of Manifolds 9. Fuzzy sets and their applications. 10. Algebraic Number theory.	Theoretical	100	3.00
7	Viva-voce	Viva-voce	100	
	Total Marks =600			

M.A./M.Sc.(Mathematics) Previous
Paper-I
Abstract Algebra

M.M.: 80 Duration:-3.00 hours

Group-Automorphisms, Inner automorphism, Automorphism groups and their computations. Conjugacy relation. Normaliser, Counting principle and the class equation of a finite group. Center for Group of prime-order. Abelianizing of a group and its universal property. Sylow's theorems P. Sylow subgroup. Structure theorem for finite Abelian gropus. Ring theory : Ring-homomorphism. Idelas and Quotient Rngs.

Field of Quotients of an intergral Domain. Euclidean Rings. Polynomial Rings. Polynomials over the Rational Field. The Fisenstein Criterion. Polynomial Rings overl Commutative Rimgs. Unique factorization domain. R. unique factorization domain implies so is R [x1, x2, ...xn]

Groups-Normal and Subnormal series. Composition Series. Jordan-Holder theorem. Solvable groups, Nilpotent groups. Field theory-Extension fields. Algebraic and transcendental extensions. Separable and inseparable extensions. Normal extensions. Perfect fields. Finite fields. Primitive elements. Algebraically closed fields. Automorphism of extensions. Galois extensions. Fundamental theorem of Galois theory. Solution of polynomial equations by radicals. Insolvability of the general equation of degree 5 by radicals.

Modules, Submodules, Quotient modules. Homomorphism and Isomorphism theorems. Cyclic modules, Simple modules. Semi-simple modules. Schuler's lemma Free modules. Noetheiran and artinan modules and rings-Hilbert basis theorem. Wedderburn-Artin theorem. Uniform modules, primary modules and Noether Lasker theorem.

References :

1. *I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.*
2. *P.B. Bhattachrya, S.K. Jain and S.R. Nagpaul, Basic Abstract Algebra, Cambridge University Press, Indian Edition, 1997.*
3. *M.Artin. Algebra, Prentice-Hall of India, 1991.*
4. *N. Jacobson, Basic Algebra, Vols. I & IIi, W.H. Freeman, 1980.*
5. *S. Lang, Algebra, Addison-Wesley, 1991.*

M.A./M.Sc.(Mathematics) Previous
Paper-II
Real Analysis

M.M.: 80 Duration:-3.00 hours

Metric Spaces : Definition and examples of metric spaces Neighbourhoods. Limit points Interies points, Open and closed sets. Closure and interior. Boundary points. Sub-space of a metric space. Cauchy sequences, Completeness. Cantor's intersection theorem. Contraction principle. Construction of real number as the completion of the incomplete metric spaces of rationals. Real numbers a complete ordered field. Dense subsets. Baire Category theorem. Separable, second countable and first countable spaces. Continuous functions. Extension theorem. Uniform continuity. Isomentry and homeoporphism. Equivalent metrics. Compactness. Sequential compactness. Totally bounded spaces. Finite intersection property. Continuous functions and compact sets. Connectedness. Components. Continuous functions and connected sets. Definition and existence of Riemann-Sieltjes integral. Properties of the integral, Integration and differentiation, the fundamental theorem of Calculus, integration of vecor-valued functions. Rectifiable Curves.

Rearrangements of terms of a series, Riemann's theorem. Sequences and series of function, pointwise and uniform convergence. Cauchy criterion for uniform, convergence, Welerstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, uniform convergence and differentiation, Weiesrtrass approximation theorem, Power series, uniqueness theorem for power series, Able's and Tauber's theorems.

Functions of Several Variable linear transformation, Derivatives, in an open subset of Rn, Chain rule, Partial derivatives, linear transformation, Derivatives in an open subset of higher orders, Thylor's theorem, Invere function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Differentiation of integrals. Partitions of unity, Differential forms, Stoke's theorem.

References :

1. *Shanti Narayan, A Course of Mathematical Analysis, S. Chand & Co., New Delhi.*
2. *T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.*
3. *Walter Rudin, Principles of Mathematical Analysis, McGraw Hill Kogakusha, 1976.*
4. *E. Hewitt and K. Stromberg, Real and Abstract Analysis, Berlin, Springer, 1969.*
5. *Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc., New York, 1975.*
6. *T.P. Natanson. Theory of Functions of Real Variable, Vol. I, Frederick Unger Publishing Co. 1961.*

M.A./M.Sc.(Mathematics) Previous
Paper-III
Topology

M.M.: 80 Duration:-3.00 hours

Countable and uncountable sets. Infinite sets and the Axiom of Choice, Cardinal numbers and its arithmetic. Schoeder-Bemstem theorem. Cantor's theorem and the continuum hypothesis. Zorn's lemma. Well-ordering theorem.

Definition and examples of topological spaces, Closed sets. Closure, Dense subsets. Neighborhoods. Interior, exterior and boundary. Accumulation points and derived sets and bases, sub-bases. Subspaces and relative topology.

Alternate methods of defining a topology in terms of Kuratowski Closure Operator and Neighborhood System. Continuous functions homeomorphism.

First and Second Countable spaces. Lindelof's theorems. Separable space second. Countability and Separability.

Separation axioms T0,T1, T2, T3, T4 ; their Characterizations and basic properties. Urysohn's lemma. Tietze extension theorem.

Compactness, Continuous functions and compact sets, Basic properties of compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification. Stone-vech compactification. Compactness in metric spaces. Equivalence of compactness, countable compactness and sequential compactness in metric spaces.

Connected spaces. Connectedness on the real line. Components. Locally connected spaces.

Tycholnoff product topology in terms of standard sub-base and its characterizations. Projection maps. Separation axioms and product spaces. Connectedness and product spaces. Compactness and product spaces (Tychonoff's theorem). Countability and product spaces.

Embedding and metrization. Embedding lemma and Tychonoff embedding. The Urysohn metrization theorem.

Reference :

1. *James R. Munkers, Topology, A First Course, Prentice-Hall of India Pvt. Ltd. New Delhi. 200.*
2. *George F. Simmons, Introduction to Topology and Modern Analysis, MeGraw Hill Book Company, 1963.*
3. *K.D. Joshi, Introduction to General Topology, Wiley Eastern Ltd. 1983.*
4. *J. Hocking and G. Young, Topology, Addison-Wesley, Reading, 1981.*
5. *W.J. Pervin, Foundations of General Topology, Academic Press Inc. New York, 1964.*

M.A./M.Sc.(Mathematics) Previous
Paper-IV
Complex Analysis

M.M.: 80 Duration:-3.00 hours

Maximum modulus principle. Schwarz lemma, Laurent's series. Isolated singularities. Meromorphic functions. The argument principal. Rouche's theorem. Inverse function theorem.

Residues, Cauchy's residue theorem. Evaluation of integrals. Branches of many valued functions with special reference to arg z, log z and za.

Bilinear transformations, their properties and classifications. Definitions and examples of Conformal mappings.

Spaces of analytic functions. Hurwitz's theorem. Montel's theorem. Riemann mapping theorem.

Weierstrass' factorization theorem. Gamma function and its properties. Riemann Zeta function. Reimann's functional equation. Runge's theorem. Mittag-Leffler's theorem. Analytic Countinuation. Uniqueness of direct analytic continuation. Uniqueness of direct analytic continuation. Uniqueness of analytic continuation along a curve. Power series method of analytic continuation. Schwarz Reflection principle. Monodromy theorem and its consequences, Harmonic functions on a disk. Haranck's nequality and theorem. Dirichlet problem. Green' function.

Cannonical product. Jensen's formula. Poisson-Jensen formula. Handamard's three circles theorem. Order of an entire function. Exponent of Convergence. Borel's theorem. Hadamard's factorization theorem.

The range of an analytic function. Bloch's theorem. The Little Picard theorem. Schottky's theorem. Montel Caratheodory and the Great picard theorem.

Univalent functions, Bieberbach's conjecture (Statement only) and the "1/4- theorem.

References :

1. *H.A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.*
2. *E.C. Titchmarsh, The Theory of Functions, Oxford University Press, London.*
3. *L.V. Ahlfors, Complex Analysis, MC Graw Hill, 1979.*
4. *S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.*
5. *Walter Rudin, Real and Complex Analysis, McGraw Hill Book Co., 1968.*
6. *E. Hille, Analytic Function Theory, Hindustan Book Agency, Delhi, 1994.*

M.A./M.Sc.(Mathematics) Previous
Paper-V
Differential Geometry

M.M.: 80 Duration:-3.00 hours

Local theory of curves-Space curves, Examples. Planar curves. Helices. Serret-Frenet apparatus, Existence of space ourves. Involutes and evolutes of curves.

Global Curve Theory- Rotation index. Convex Curves. Isoperimetric inequality. Four vertex theorem.

Local Theory of Surfaces- Parametric patches on surface. First Fundamental form and are length. Normal curvature. Goedesic curvature and Gauss formulae. Shape operator Lp of a surface at a point. Vector field along a curve. Second and third fundamental forms of a surface. Welengarten map. Principal curvatures. Gaussian Curvature. Mean and normal curvatures. Gauss theorem egregium. Isometry groups and the fundamental existence theorem for surfaces.

Global Theory of surfaces- Geodesic coordinate patches. Gauss- Bonnet formulae. Euler characteristic of a surface. Index of a vector field. Spaces of constant curvature.

Intrinsic Theory of Surface in Riemannian Geometry- Parallel transition and connections, Cartan's structural equations and curvature. Interpretation of curvature. Geodesic curvature and Gauss-Bonnet for a 2-dimensional Riemann surface. Geodesic coordinate systems. Isometries and spaces of constant curvature and the-3 types of geometry.

Transic Extension Theory of surfaces in R3- Spherical image. Parallel translation for imbedded surfaces in R3- Classification of compact connected oriented surfaces in R3- relative to curvature.

Elements of general Riemannian Geometry- Concepts of manifolds and examples. Riemannian metric. Tensor fields. Covarient differentiation. Symmetry properties of curvature tensor. Concept of affine connection. Christoffel symbols. Curvature and torsion tensors. Riemannian metric and affine connection geodesic and normal coordinates. Fundamental theorem of Riemannian geometry.

References :

1. *J.A. Thorpe, Introduction to Differential Geometry, Springer-verlge.*
2. *B.O.' Neill, Elementray Differential Geometry, Academic Press, 1966.*
3. *M. DoCarno, Differential Geometry of curves and surfaces, Prentice-Hall, 1976.*
4. *T.J. Willmore, An Introduction to Differential and Riemannian Geometry, Oxford University Press, 1965.*
5. *D. Laugwitz, Differential and Riemannian Geometry, Academic Press, 1965.*

Optional

Paper-VI

Group-A

Differential Equations

M.M.: 100 Duration:-3.00 hours

Preliminaries- Initial value problem and the equivalent integral equation, mth order equation in d-dimensions as a first order system, concepts of local existence, existence in the large and uniqueness of solutions with examples.

Basic Theorems- Ascoli- Arzela Therorem. A theorem on convergence of solutions of a family of initial values problems. Picard- Lindel of theorem- Peano's existence theorem and corollary. Maximal intervals of existence. Extension theorem and corollaries. Kamke's convergence theorem. Kneser's theorem (statement only).

Differential Inequaliies and Uniqueness- Gronwall's inequality. Maximal and Minimal solutions. Differential inequalities. A Theorem of Winter. Uniqueness Theorems. Naguma's and Osgood's criteria.

Egres points and Lyamunov functions. Successive approximations.

Linear Differential Equations- Linear Systems. Variation of constants, reduction to smaller systems. Basic inequalities, constant coefficients. Floquet theory. Adjoint systems. Higher order equations.

Dependence on initial conditions and parameters; Preliminaries. Continuity. Differentiability. Higher Order Differentiability.

Poincare- Bendixson Theory- Autonomous systems. Umlanfsatz, Index of a stationary point.

Poincare- Bendixson theorem. Stability of periodic solutions, rotation points. Foce, nodes and saddle points. Linear second order equations- Preliminaries Basic facts. Theorems of Sturm. Sturm-Liouville Boundary Value problems. Number of zeros. Nonsociallatory equations and principal solutions. Nonoscillation theorems.

Use of Implicit function and fixed point theorems-Periodic solutions. Linear equations. Nonlinear problems.

Second order Boundary value problems- Linear problems.

Nonlinear problems, Aprorri bounds.

Recommended Text :

P. Hartman, Ordinary Differential Equations, John Wiley (1964).

References :

1. *W.T. Reid, Ordinary Differential Equations, John Wiley & Sons, NY, 1971.*
2. *E.A. Coddington and N. Levinson, Theory of Ordinary Differential Equations. McGraw-Hill, NY, 1955.*

Group-B

Advanced Discrete Mathematics

M.M.: 100 Duration:-3.00 hours

Formal Logic-Statements. Symbolic Representation and Tautologies, Quantifiers. Predicates and Validity. Propositional Logic.

Semigroups & Monoids- Definitions and Examples of Semigroups and Monoids (including those pertaining to concatenation operation). Homomorphism of semigroups and monoids. Congruence relation and Quotient Semigroups. Subsemigroup and submonoids. Direct product. Basic Homomorphism Theorem.

Lattices-Lattices as partially ordered sets. Their properties. Lattices as Algebraic systems. Sublattices. Direct product, and Homomorphisms. Some Special Lattices e.g., Complete, Complemented and Distributive Lattices.

Boolem Algebras- Boolean Algebras as Lattices. Various Boolean indentities. The switching Algebra example. Subalgebras, Direct Product and Homomorphisms, Joinrreducible elements, Atoms and Minterms. Boolean Forms and their Equivalence. Minter Boolean Forms. Sum of products. Cannonical Forms. Minimization of Boolean Functions, Application of Boolean Algebra to Switching Theory (using AND, OR & NOT gates). The Karnaughj Map method.

Graph Theory- Definition of (undirected) Graphs, Paths, Circuits, Cycles & Subgraphs, Induced Subgraphs. Degree of vertex. Connectivity. Planar Graphs and their properties. Thress. Euler's Formula for connected Planar Graphs. Complete & Complete Bipartite Graphs. Kuratowski's Theorem (statement only) and its use. Spanning Trees. Cut-sets, Fundamental Cut-sets, and Cycles. Minimal Spanning Trees and Kruskal's Algorithm. Matrix Representation of Graphs. Euler's Thorem of a Vertex. Weighted undirected Graphs. Oijkstra's Algorithm. Strong Connectivity & Warshall's Algorithm. Directed Tress. Search Trees. Tree Traversals.

Introductory Computability Theory- Finite State Machines and their Transition table Diagrams. Equivalence of Finite State Machines. Reduced Machines. Homomorphis. Finite Automata. Acceptors. Non-deterministic Finite Automata and equivalence of its power to that of Deterministic Finite Automata. Moore and Mealy Machines.

Turing Machine and Partial Recursive Functions.

References :

1. *C.L. Liu, Elements of Discrete Mathematics, McGraw-Hill Book Co.*
2. *S. Wiitala, Discrete Mathematics- A Unfired Approach, McGraw-Hill Book Co.*
3. *J.E. Hopcroft and J.D. Ullaman, Introduction to Automata Theory, Languages & Computation, Narosa Publishing House.*
4. *J.L. Gersting, Mathematical Structures of Computer Science. Computer Science Press, New York.*

Group-C

Spherical Astronomy

M.M.: 100 Duration:-3.00 hours

Refraction. Parallax. Aberration. Precession and Nutaion. Eclipses. The proper motion of stars. Astronomical photography determination of positions. Binary Stars. Occulation.

References :

1. *G. Prasad, Spherical Astronomy*
2. *Ball, Astronomy*
3. *Smart, Spherical Astronomy*

Group-D

Special Theory of Relativity

M.M.: 100 Duration:-3.00 hours

Review of Newtonian mechanics- Intertial frames. Speed of light and Galilean relativity. Michelson-Morley experiment. Lorentz-Fitzerold contraction hypothesis. Relative character of space and time. Postulates of special theory of relativity. Lorentz transformation equations and its geometrical interpretaton. Group properties of Lorentz transformations. Relativistic Kinematics- Composition of parallel velocities. Length contraction. Time dilation. Transformation equation equations for components of velocity and accelaeration of a particle and Lorentz contraction factor.

Geometrical representation of space-time-Four dimensionla Minkowskian space-time of special relativity. Time – like, light – like and space – like intervals. Null cone, Proper time. World line of a particle. Four vectors and tensors in Mindowskian space-time.

Relativistic mechanics- Variation of mass with velocity. Equivalence of mass and energy. Transformation equations for mass momentum and energy. Energy-momentum for vector. Relativistic force and Transformation equations for its components. Relativistic Lagrangian and Hamiltonian. Relativistic equations of motion of a particle. Energy momentum tensor of a continuous material distribution. Electromagnetism- Maxwell's equations in vacuo.

Transformation equation for the densities of electric charge and current. Propogation of electric and magnetic field strengths. Transformation equations for electromagnetic four potential vector. Transformation equations for electric and magnetic field strengths. Gauge transformation. Lorentz invariance of Maxwell's equations. Maxwell's equations in tensor form. Lorentz force on a charged particle. Energy momentum tensor of an electromagnetic field.

References :

1. *C. Moller, The Theory of Relativity, Oxford Clarendon Press, 1952.*
2. *J.L. Anderson, Principles of Relativity Physics, Academic Press, 1967.*
3. *W. Rindler, Essential Relativity, Van Nostrand Reinohold Company, 19693.*
4. *R. Resnick, Introduction to Special RElativyt, Wiley Eastern Pvt. Ltd. 1972.*

Group-E

Integral Equation and Boundary Value Problems.

M.M.: 100 Duration:-3.00 hours

Classification of integral equations of Volterra and Fredholm types; Conversion of intitial and boundary value problem into integral equation; Conversion of integral equation into differential equation (When it is possible); Volterra and Fredholm integral operators and their iterated kernels; Resolvent kernels and Neumann series method for solution of integral equations; Branch contraction principle, its application in solving integral equations of second kinds by the method of successive interation and basic existence theorem; Abel intgral equation and tantochrone problem; Fredholm-alternative for Fredholm integral equation of second kind with degenerated kernels; Use of Laplace and Fourier Transform to solve integral equations.

Definition of a boundary value problem for an ordinary differential equation of the second order and its reduction to a fredholm integral equation of the second kind; Dirac Delta Function; Green Function for ordinary differential initial and boundary value problem.

References :

1. *R.P. Kanwai, Linear Integral Equation : Theory and Techniques. Academic Press, New Youk, 1971*
2. *S.G. Mikhin, Linear Integral Equation. Hindustan Book Agency, 1960.*
3. *I.N. Sneddon. Mixed Boundary Value Problem in Potential Theory. North HOLLAND, 1966.*
4. *Shanti Shwaroop. Linear Integral Equation. Krishna Prakashan, Meerut.*

M.A./M.Sc. Mathematics (Final)

Paper-I

Measure and Integration

M.M.: 75 Duration:-3.00 hours

Lebesgue outer measure. Measureable sets. Regularity. Measureable functions. Borel and Lebesuge measurability. Non-measurable sets.

Integration of Non-negative functions. The General integral, Integration of Series. Reimann and Lebesgue integrals. The Four derivatives. Functions of Bounded variation. Lebesuge Differentiation Theorem. Differentiation and Integration.

Measures and outer measures. Extension of measure. Uniquenss of Extension. Completion of a measure. Measure spaces. Integration with respect to a measure.

The Lp-spaces. Convex functions. Jensen's inequality. Holder and Mindowski inequalities. Completeness of Lp. Convergence in Measure. Almost uniform convergence.

Signed measure. Hahn docomposition theorem, mutully singular measures. Radon-Nikodym theorem. Lebesgue decomposition. Riesz representation theorem. Extension theorem (Caratheodory). Lebesgue- Stieltjes integral, product measures, Fubini's theorem. Differentitation and Integration. Decomposition into absolutely continuous nd singular parts. Baire sets. Baire measure, continuous functions with compact support. Regularity of measures on locally compact spaces. Integration of continuous functions with compact support. Riesz-Markoff theorem.

References :

1. *P.K. Jain and V.P. Gupta, Lebesgue Measure and Integration, New Age International (P) Limited Published, New Delhi, 2000.*
2. *J.H. Williamson, Lebesgue Integration, Holt Rinehart and Winston, Inc. New York, 1962.*
3. *P.R. Halmos, Meausure Theory, Van Nostrand, Princeton, 1950.*
4. *Inder P. Rana, An Introduction to Measure and Integration, Narosa Publishing House, New Delhi, 1997.*
5. *G. de Barra, measure Theory and Integration, Wiley Eastern Ltd., 1981.*

M.A./M.Sc. Mathematics (Final)
Paper-II
Partial Differential Equations

M.M.: 75 Duration:-3.00 hours

Calculus of Variation-Variational Problem with Moving Boundaries-Functionals dependent on one and two functions. One sided variations.

Sufficient conditions for an Extremum-Jacobi and Legendre conditions. Second Variation. Variational Principle of least action.

Example of PDE. Classification.

Transport Equation- Initial value Problem. Non-homogeneous Equation.

Laplace's equation- Fundamental solution. Mean Value Formula Properties of Harmonic Functions. Green's Function. Energy Methods.

Heat Equation- Fundamental Solution. Mean Value Formula.

Properties of Solutions Energy Methods.

Wave Equation- Solution by Spherical Mean, Non-homogeneous Equations. Energy Methods.

Nonliner First Order PDE-Complete Inegrals, Envelopes, Characteristics. Hamilton Jacobi Equations (Calculus of Variations. Hamilton's ODE. Legendre Transform. Hopf-Lax Formula. Weak Solutions. Uniqueness). Conservation Laws (Shocks, Entropy Condition, Lax-Oleink Formula, Weak Solutions, Uniqueness, Riemann's Problem. Long Time Behaviour).

Representation of Solutions- Separation of Variables. Similarity Solutions (Plane and Travelling Waves. Solitions, Similarity under Scaling), Fourier and Laplace Transform. Hopf-Cole Transform, Hodography and Legendre Transfroms, Potential Functions, Asymptotic (Singular Perturbations, Laplace's Method. Geometric Optics. Stationary Phase, Homogenization). Power Series (Non-charcteristic surface. Real Analytic Functions, Cauchy-Kovalevskaya Theorem).

References :

1. *I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall.*
2. *A.M. Arthurs, Complementary Variations Principle, Clarendon Press, Oxford, 1970.*
3. *A.s. Gupta, Calculus of Variations with Applications, Prentice Hall of India, 1997.*
4. *L.C. Evans, Partial Differential Equatins, Graduate Studies in Mathematics, Volume 19, AMS, 1998.*
5. *I.N. Sendon, Elements of Partial Differential Equations, McGraw Hill Book Co., 1988.*
6. *P. Prasad and R. Ravindran, Partial Differential Equations.*

M.A./M.Sc. Mathematics (Final)

Paper-III

Mechanics

M.M.: 75 Duration:-3.00 hours

Rotation of a vector in two and three dimensional fixed frame of refrence. Kinetic energy and angular momentum of rigid body rotating about its fixed point.

Euler dynamical and geometrical equations of motion.

Analytical Dynamics

Generalized coordianates. Holonomic and Non-holonomic systems. Scleronomic and Rheonomic systems. Generalized potential. Lagrange's equation of first kind. Lagrange sequations of second kind. Uniqueness of solution. Energy equation for conservative fields.

Hamilton's variables Donkin's theorem. Hamilton canonical equations. Cyclic coordinates Routh's equations. Poisson's Bracket. Poisson's Identity. Jacobi-Poisson Theorem. Motivating Problems of calculus of variations. Shortest distance. Minimum surface of revolution. Brachistochrone problem. Isoperimetric problem. Geodesic Fundamental lemma of calculus of variation. Euler's equation for one dependent function and its generalization to (1) n dependent functions (2) higher order derivatives. Conditional extremum under geometric constraints and under integral constraints.

Hamilton's Principle. Principle of least action. Poincare Cartan Integral invariant. Whittakers equations. Jacobi's equations. Statement of Lee Hwa Chung's theorem.

Hamilton-Jacobi equation. Jacobi theorem. Method of separation of variables. Lagrange Brackets. Condition of canonical character of a transformation in terms of Lagrange brackets and Poisson brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations.

References :

1. *H. Goldstein, Classical Mechanics, Narosa Publishing house, New Delhi.*
2. *A.S. Ramsey, Dynamics Part II, The English Language Book Society and Cambridge University Press, 1972.*
3. *F. Gantmacher, Lectures in Analytic Mechanics, MIR Publishers, Moscow, 1975.*
4. *Narayan Chandra Rana & Promod Sharad Chandra Joag, Classical Mechanics, Tata McGraw Hill, 1911.*

M.A./M.Sc. Mathematics (Final)

Paper-IV

Functional Analysis

M.M.: 75 Duration:-3.00 hours

Normed and Banch speces- Definitions and elementry properties. Some comcrete normed and Banch spaces. Quogient spaces. Completion of normed spaces.

Bounded linear operators- definitions, examples and basic properties. Spaces of bounded linear operators. Equivalent norms. Finite dimensional normed spaces and copmactedness. Open mapping theorem and its consequences. Closed graph theorem and its consequences. Uniform boundedness principal.

Bounded linear functionals- definitions, examples and basic properties. The form of some dual spaces. Hahn-Banch theorem and its consequences. Embedding and reflexivity of normed spaces. Adjoint of bounded linear operators. Weak conversions and weak* conversions.

The concept and specific geometry of Hilbert spaces-Definitions and basic properties of inner product spaces and Hibert spaces. Completion of inner product spaces. Orthogonality of vectors Orthogonal complements and projection theorem. Orhtonormal sets and Fourier analysis complete Orthonoraml sets.

Functional and operators on Hilbert spaces-bouded linear functionals. Hilbert-adjoint operators. Self-adjoint operators. Normed operators. Unitary operators. Orhtogonal projection operators.

Book Prescribed :

P.K. Jain, O.P. Ahuja and Khalil Ahmad, Functional analysis, New Age International (P) Ltd., Lucknow.

Book Recommended :

1. *K.K. Jha, Functional Analysis, Students Friends. 1986.*
2. *A.H. Siddiqi, Functional Analysis with applications. Jata Mc Graw Hill Publishing Company Ltd, New Delhi.*
3. *Walter Rudin, Functional Analysis, Jata Mc Graw Hill Publishing Co. Ltd., New Delhi 1973.*

Two optional papers

Paper-V & Paper-VI

Two papers out of the following have to be chosen keeping in view the prerequisite and suitability of the combination.

1.

Fluid Mechanics

M.M.: 100 Duration:-3.00 hours

Kinematics- Lagrangian and Eulerian methods. Equations of continuity. Boundary surfaces. Stream lines. Path lines and streak lines. Velocity potential. Irrotaional and rotational motions. Vortex lines.

Equations of Motion- Lagrange's and Euler's equations of motion. Bernoulli's theorem. Equations of motion by flux method. Equations referred to moving axex. Implulsive actons. Stream function. Irrotaional motion in two-dimensions. Complex velocity potential. Sources, sinks, doublets and their images. Conformal mapping. Miline- Thomson circle theorem.

Two-dimensional irrotational motion produced by motion of circular, co-axial and elliptic cylinders in an infinite mass of liquid. Kinetic energy of liquid. Theorem of Balsius. Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere. Equation of motions of a sphere. Stoke's stream function.

Vortex motion and its elementary properties. Kelvin's proof of permances. Motions due to circular and rectilinear votices. Wave motion in a gas. Speed of Sound. Equation of motion of a gas. Subsonic, sonic and supersonic flows of a gas. Isentropic gas flows. Flow through a nozzle. Normal and oblique schocks.

Stress components in a real fluid. Relations between rectangular components of stress. Connection between stresses and gradients of velocity. Navier-stoke's equations of motion. Plane Poiseuilee and Couette flows between two parallel plates. Theory of Lubrication. Flow through tubes of uniform cross section in from of circle, annulus, ellips and equilateral triangel under constant pressure gradient. Unsteady flow over a flat plate.

Dynamical similarity. Buckighm p-theorem. Reynolds number. Prandt's boundary laryer. Boundary layer equations in two-dimensions. Blasius solution. Boundary layer thickness. Displacement thickness. Karman integral conditions. Separation of boundary layer flow.

References :

1. *W.H. Besaint and A.S. Ramsey, A Treatise on Hydromechanics Part Ii, CBS Publisher, Delhi, 1988.*
2. *G.K. Batchelor, An Introduction to Fluid Mechanics, Foundation Books, New Delhi, 1994.*
3. *F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers, Delhi, 1985.*

2.

Difference Equations

M.M.: 100 Duration:-3.00 hours

Introduction, Difference Calculus-The Difference Operator, Summation, Generating functions and approximate summation.

Linear Difference Equations- First order equations. General results for linear equations. Equations with constant coefficients. Applications. Equations with variable coefficients. Nonlinear equations that can be linerized. The z-transform

Stability Theory- Initial value problems for linear systems. Stability of linear systems. Stability of nonlinear systems, Chaotic behaviour.

Aysmptotic methods- Introduction. Asymptotic analysis of sums. Linear equations. Nonlinear equations.

The self-adjoint second order linear equation. Introduction. Sturmian Theory. Green's functions. Disconjugacy. The Riccati Equation. Oscillation.

The Strum-Liouville problem- Introduction, Finite Fourier Analysis. A non-homogeneous problem.

Discrete Calculus of variations- Introduction. Necessary conditions. Sufficient Conditions and Discounjugacy.

Boundary Value Problems for Nonlinear equations- Introduction. The Lipschitz case. Existence of solutions. Boundary Value Problems for Differential Equations.

Partial Differential Equations.

Discretization of Parial Differential Equations.

Solution of Partial Differential Equations.

Recommended Text.

Walter G. Kelley and Allan C. Perterson- Difference Equations. An INtroduction with Applicatins. Academic Press Inc. Harcourt Brace Joranovich Publishers, 1991.

References :

Calvin Ahlbrandt an Allan C. Peterson, Discrete Hamiltonian Systems, Difference Equations, Continued Fractions and Ricati Equations, Kluwer, Boston, 1996.

3.

Mathematics of Finance

M.M.: 100 Duration:-3.00 hours

Financial Derivatives- An Introduction; Types of Financial Derivatives- Forwards and Futures; Options and its kinds; and SWAPS.

The Arbitrage Theorem and Introduction to Portfolio Selection and Capital Market Theory : Static and Continuous- Time Model.

Pricing by Arbitage- A Single- Period option Princing Model; Multi- Period Pricing Model- Cox- Ross- Rubinstein Model; Bounds on Option Prices.

The Ito's Lemma and Ito's Integral.

The Dynamics of Derivative prices-Stochastic Differential Equations (SDEs)- Major Models of SEDs; Linear Costant Coefficient SDEs; Geometric SDEs' Square Root Process; Mean Reverting Process and Omstein-Uhlenbeck Process. Martingle Measures and Rick-Neurtral Probabilities; Pricing of Binomial Options with equivalent martingate measures.

The Black- Scholes Option Pricing Model- using no arbitrage approach, limiting case of Binomial Option Princing and Risk- Neutral probabilities.

The American Option Princing- Extended Trading Strategies; Analysis of American Put Options; early exercise premium and relation to free boundary problems.

Reference :

1. *Sheldon M. Ross, An Introduction to Mathematical Finance, Cambridge University Press.*
2. *Salih N. Neftci, An Introduction to the Mathematics of Financial Derivatives, Academic Press, Inc.*
3. *Robert J. Elliot and P. Ekkehard Kopp, Mathematics of Financial Markets, Springer- Verlag, New York Inc*

4.

Information Theory

M.M.: 100 Duration:-3.00 hours

Measure of Information- Axioms for a measure of uncertainty. The Shannon entropy and its properties. Joint and conditional entropies. Transformation and its properties. Noiseless coding- Ingredients of noiseless coding problem. Uniquely decipherable codes. Necessary and sufficient condition for the existence of instantaneous codes. Construction of optimal codes.

Discrete Memoryless Channel- Classification of channels. Information processed by a channel. Calculation of channel capacity. Decoding schemes. The ideal observer. The fundamental theorem of Information theory and its strong and weak converses.

Continuous Channels- The time-discrete Gaussian channel. Uncertainty of an absolutely continuous random variable. The converse to the coding theorem for time-discrete Gaussian channel. The time-continuous Gaussian channel. Band-limited channels.

Some imuitive properties of a measure of entropy-Symmetry, normalization, expansibility, boundedness, recursivity, maximality, stability, additivity, subadditivity, nonnegativity, countinuity, branching etc. and interconnections among them. Axiomatic characterization of the Shanon entropy due to Shannon and Fadeev.

Information functions, the fundamental equation of information, information functions continuous at the origin, nonnegative bounded information functions, measurable information functions and entropy. Axiomatic characterizations of the Shannon entropy due to Tverberg and Leo. The general solution of the fundamental equation of information. Derivatives and their role in the study of information functions.

The branching property. Some characterizations of the Shannon entropy based upon the branching property. Entropies with the sum property. The Shannon inequality. Subadditive, additive entropies.

The Renji entropies, Entrpies and mean values. Average entropies and their equality, optimal coding and the Renji entropies. Characterization of some measures of average code length.

References :

1. *R. Ash, Information Theory, Interscience Publishers, New York, 1965.*
2. *F.M. Reza, An Introduction to Information Theory, McGraw-Hill Book company Inc. 1961.*
3. *J. Aczel and Z. Daroczy, On measures of information and their characteization.*

5.

Algebraic Topology

M.M.: 100 Duration:-3.00 hours

Fundamental group functo, homotopoy of maps between topological spaces, homotopy equivalence, contractible and simply connected spaces, fundamental groups of S^1, and S^2 x S^1 ect.

Calculation of fundamental group of S^n, n>1 using Van Kampen's theorem, fundamental groups of a topological group, Brouwer's fixed point theorem, fundamental theorem of algebra, vector fields on planer sets, Frobenius theorem for 3x3 matrics.

Covering spaces, unique path lifting theorem, covering homotopy theorems, group of covering transformations, criterion of lifting of maps in terms of fundamental groups, universal covering, its existence, special cases of manifolds and topological groups.

Sigular homology, reduced homology, Eilengerg Steenrod axioms of homology (no proof for homotopy invariance axion, excision axiom and exact sequence axiom) and their application, relation between group and first homology.

Calculation of homology of Sn, Brouwer's fixed point theorem for F: E^n E^n, application spheres, vector fields, Mayer-Vietories sequence (without proof) and its applications.

Mayer Victories sequence (withh proof) and its application to calculation of homology of graphs, torus and compact surface of genus g, collared pairs, construction of spaces by attaching of cells, spherical complexes with examples of S^n, r-leaved rose, torus, **RP^n, CP^n** etc.

Computation of homology of **CP^n, RP^n,** torus, suspension space XVY, vompact surface of genus g and non-orientable surface of genus h using Mayer Vietoris sequence, Betti numbers and Euler characteristics and their calculation for Sn, r-reveled rose, **RP^n, CP^n, S^2 x S^2, X+Y,** etc.

Singular cohomology modules, modules, Kronecker product, connection homomorphism, contra functionality of singular cohomology modules, naturality of connecting homomorphism, exact chomology sequence of pair, homotopy invariance, excision properties, chomology of a point. Mayer Vietoris sequence and its application in computation of chomology of **S^n, RP^n, CP^n,** torus, compact surface of genus g and non-orientable compact surface.

Compact connected 2- manifolds, their orientability and non-orientability, examples, connected sum, construction of projective space and Klein's bottle from a square, Klein's bottle as union of two Mobius strips, canonical form of sphere, torus and projective planes, Kelin's bottle. Mobius strip, triangulation of compact surfaces.

Classification theorem for compact surfaces, connected sum of torus and projective plane as the connected sum of tree projective planes, Euler characteristic as a topological invariant of compact surface, connected sum formula, 2- manifolds with boundary and their classifications, Euler characteristic of a bordered surface, models of compact bordered surfaces in **R^3**.

References :

1. *James R. Munkres, Topology- A first Course, Prentice Hall of India Pvt. Ltd., New Delhi 1978.*
2. *Marwin J. Greenberg and J.R. Harper, Algebraic Topology-A first Course, Addison- Wesley Publishing Co., 1981.*
3. *W.S. Massey, Algebraic Topology- An Introduction, Harcourt, Brace and World Inc. 1967, SV., 1977.*

6.

Operations Research

M.M.: 100 Duration:-3.00 hours

Operations Research and its Scope, Necessity of Operations Research in Industry.

Linear Programmin-Simplex Method. Theory of the Simplex Method. Duality and Sensitivity Analysis.

Other Algorithms for linear Programming- Dual Simplex Method. Parametric Linear Programming. Upper Bound Technique. Interior Point Algorithm. Linear Goal Programming.

Transportation and Assignment Problems.

Network Analysis- Shortest path problem. Minimum Spanning Tree problem. Maximum Flow Problem. Minimum Cost Flow Problem. Network Simplex Method. Project Planning and Control with PERT-CMP.

Dynamic Programmin- Deterministic and Probabilistic Dynamic Programming.

Game Theory- Two Person, Zero- Sum Games. Games with Mixed Strategies. Graphical Solution. Solution by Linear Programming.

Integer programming- Branch and Bound Technique.

Application to Industrial Problems- Optimal product mix and activity levels. Petroleum-refinery operations. Blending problems. Economic interpretation of dual linear programming problems. Input-Outpur analysis. Leontief system. Indecomposable and Decomposable exconomies.

Nonlinear Programming- One and Multi-Variable Unconstrained Optimization. Kuhn-Tucker. Conditions for constrained Optimization. Quadract Programming Separable Programming. Convex Programming. Non-convex Programming.

References :

1. *H.A. Taha, Operations Research- Ann Introduction, Macmillan Publishing Co. Inc., New York.*
2. *N.S. Kambo, Mathematical Programming Techniques, Affiliated East- West Press Pvt. Ltd. New Delhi, Madras.*
3. *S.S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd. New Delhi.*
4. *Prem Kumar Gupta and D.S. Hira, Operations Research- An Introductions. S. Chand & Co. Ltd., New Delhi.*
5. *F.S. Hillier and G.J. Lieberman, Introduction to Operations* Research, McGraw Hill International Edition, Industrial Engineering Series, 1995.

7.

General Relativity and Cosmology

M.M.: 100 Duration:-3.00 hours

General Relativity- Transformation of coordinates. Tensors. Algebra of Tensors. Symetric and skew symmetric Tensors. Contraction of tensors and quotient law.

Reimannian metric, Parallel transport, Christoffel Symbols. Covariant derivatves. Intrinsic derivatives and geodesics, Reiemann Christoffel curvature tensor and its symmetry properties. Bianchi identities and Einstein tensor.

Review of the special theory of relativity and the Newtonian Theory of gravitation. Principle of equivalence and general covariance, geodesic principle. Newotonian approximation. Schwarzchild external solution and its isotropic form. Planetary orbits and analogues of Kepler's laws in general relativity. Adavance or perihelion of a planet. Bending of light rays in gravitational field. Gravitational redshift of spectral lines. Reader echo delay.

Energy- momentum tensor of a perfect fluid. Schwarzchild internal solution. Boundary conditions. Energy momentum tensor of an electromagnetic filed. Eistein-Maxwell equations. Reissner-Nordstrom solution.

Cosmology- Mach's principal. Einstein modified field equations with cosmological term. Static Cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe.

Hubble's law. Cosmological principle's Wey'Is postulate. Derivation of Robertson-Walker metric. Hubble and deceleration parameters. Redshift. Redsshift versus distance relation. Anuglar size versus redshift relation and source counts in Robertson- Walker space-time.

Friedmann models. Fundamental equations of dynamical cosmology. Critical density. Closed and open Universes. Age of the universe. Matter dominated era of the universe. Einstein-de-Sitter model. Particle and even horizons.

Eddington-Lamaitre models with I-trem. Perfect cosmological principle. Steady state cosmology.

Reference

1. *C.E. Weatherburn An Introduction to Riemannian Geometry and the tensor Calculus, Cambridge University Press, 1950.*
2. *J.V. Narlikar, General Relativity and Cosmilogy, The Machmillann Company of India Ltd. 1978.*
3. *B.F. Shutz, A first course in genral relativity, Combridge University Press, 1990.*
4. *A.S. Eddington, The Mathematical Theory of Relativity, Cambridge University Press, 1965.*
5. *S. Weinberg Gravitation and Cosmology : Principle and applications of the general theory of relativity, John Wiley & Sons, Inc. 1972.*
6. *J.V. Narlikar, Introduction to Cosmology, Cambridge University Press, 1993.*

8.

Differential Geometry of Manifolds

M.M.: 100 Duration:-3.00 hours

Definition and examples of differentiable manifolds. Tangent spaces. Jacobian map. One parameter group of transformations. Lie derivatives. Immersions and imbeddings. Distributions. Exterior algebra. Exterior derivative.

Topological groups. Lie groups and lie algebras. Product of two liegroups. One parameter subgroups and exponential maps. Emaples of liegroups Homomorphism and isomorphism. Lie transformation groups. General linear groups. Principal fibre bindle. Linear frame bundle, Associated fibre bindle. Vector bundle. Tangent bundle. Induced bundle. Bundle homomorphisms.

Riemannian mainifolds. Riemannian connection, Curvature tensor. Sectional Curvature. Schur'stheorem. Geodesics in a Riemannian manifold. Projective curvature tensor. Conformal curvature tensor.

Submainfolds & Hypersurfaces. Normals. Gauss' formulas. Weingarten equatuions. Lines of curvature. Generalized Gauss and Minardi- Cobazzi equations.

Almost Complex manifolds. Nijenhuis tensor. Contravariant and covariant almost analtytic vector fields. F-connection.

Reference

1. *R.S. Mishra, A course in tensors with applications to Riemannian Geometry, Pothishala (Pvt) Ltd., 1965.*
2. *R.S. Mishra, Structures on a differentiable manifold and their applications, Chandrama Prakshan, Allabad 1984.*
3. *B.B. Sinha, An Introduction to modern Differential Geometry, kalyani Publishers, New Delhi, 1982.*
4. *K. Yano and M. Kon Structure of Manifolds, World Scientific Publishing Co. Ltd. 1984.*

9.

Fuzzy sets and their applications

M.M.: 100 Duration:-3.00 hours

Fuzzy Sets-Basic definitions, □ level sets. Convex fuzzy sets. Basic operations on fuzzy sets. Types of fuzzy sets. Cartesian products. Algebraic products. Bounded sum and differences. T-norms and t-conorms.

The Extension Principle- The Zadeh's extension principal. Image and iverse image of fuzzy sets. Fuzzy numbers. Elements of fuzzy arithmetic.

Fuzzy relations and Fuzzy Graphs- Fuzzy relations on fuzzy sets. Composition of fuzzy relations. Min-Max composition and its properites. Fuzzy equvalence relations. Fuzzy compatibility relations. Fuzzy relations.

10.

Algebraic Number theory

M.M.: 100 Duration:-3.00 hours

Algebraic number fields and their rings of integers; calcilations for quadratic and cubic cases. Localization, Galois extensions, Dedekind rings, discrete valuation rings, completion, unramified and ramiified extensions, different discriminant, cyclotomic fields, roots of unity. Class group and the finiteness of the class number. Dirichlet unit theorem, Pell's equation. Dedekind and Riem, ann zeta functions, analytic class number formula.

References

1. *S. Lang Algebraic Number theory, GTM Vol. 110, Springer-Verlage, 1994.*
2. *J.P. Serre, Local fields, GTM Vol. 67, Springer-Verlag, 1979.*
3. *J. Esmonde, and M. Ram Murty, Problems in Algebraic Number Theory, GTM Vol. 190, Soringer-Verlag, 199.*