# V.B.S. PURVANCHAL UNIVERSITY, JAUNPUR, U.P., INDIA

## Course Structure: M.Sc. (Mathematics)

**Department of Mathematics, Prof. Rajendra Singh (Rajju Bhaiya)**
**Institute of Physical Sciences for Study and Research**
**Veer Bahadur Singh Purvanchal University, Jaunpur, U.P., INDIA**

| **SEMESTER-I** | | | |
|---|---|---|---|
| **Course Code** | **Title** | **Credits** | **Marks** |
| MAT101 | Algebra | 4 | 100 |
| MAT102 | Point-Set Topology | 4 | 100 |
| MAT103 | Complex Analysis | 4 | 100 |
| MAT104 | Elementary Differential Geometry | 4 | 100 |
| MAT105 | Classical Mechanics | 4 | 100 |

| **SEMESTER-II** | | | |
|---|---|---|---|
| **Course Code** | **Title** | **Credits** | **Marks** |
| MAT201 | Module Theory | 4 | 100 |
| MAT202 | Integral Equations and Partial Differential Equations | 4 | 100 |
| MAT203 | Mathematical Methods | 4 | 100 |
| MAT204 | Measure and Integration | 4 | 100 |
| MAT205 | Discrete Mathematics | 4 | 100 |

| **SEMESTER-III** | | | |
|---|---|---|---|
| **Course Code** | **Title** | **Credits** | **Marks** |
| MAT301 | Functional Analysis | 4 | 100 |
| MAT302 | Theory of Ordinary Differential Equations | 4 | 100 |
| MAT303 | Galois Theory | 4 | 100 |
| MAT304 | Differentiable Manifolds | 4 | 100 |
| MAT305 | Fluid Mechanics | 4 | 100 |

| **SEMESTER-IV** | | | |
|---|---|---|---|
| **Course Code** | **Title** | **Credits** | **Marks** |
| MAT401 | Wavelets | 4 | 100 |
| MAT402<br>OR<br>MAT403 | Linear Programming<br>OR<br>Riemannian Geometry | 4 | 100 |
| MAT404<br>OR<br>MAT405 | Algebraic Number Theory<br>OR<br>Non-Linear Analysis | 4 | 100 |
| MAT406<br>OR<br>MAT407 | Advanced Fluid Mechanics<br>OR<br>Representation Theory of Finite Groups | 4 | 100 |

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|---|---|---|---|
| MAT408 | Dissertation | 4 | 100 |
| MAT409 | Viva-Voce | 4 | 100 |

## M.SC. (Mathematics) Semester-I

### Paper I: MAT 101 (Algebra)

**Unit I**

Action of a group *G* on a set *S*, Examples, Stabilizer (Isotropy) subgroups and Orbit decomposition, Class equation, Translation and conjugation actions, Transitive and effective actions, Burnside theorem, Core of a subgroup.

**Unit II**

*p*-groups, Sylow subgroups, Sylow's theorems, Examples and applications, Structure of groups of order *pq*, Characterization of finite Abelian groups and finite cyclic groups of specific orders in terms of Sylow subgroups.

**Unit III**

Normal series and composition series, Schreier's refinement theorem, Zassenhaus' lemma, Jordan-Hölder's theorem, Descending chain conditions (D.C.C.), Ascending chain conditions (A.C.C.), Examples, Internal and External direct products and their relationship, In decomposability.

**Unit IV**

Commutator or derived subgroup, Commutator series, Solvable groups, Solvability of subgroups and factor groups and of finite *p*-groups, Examples, Lower and upper central series, Nilpotent groups and their equivalent characterizations.

**Unit V**

Prime and irreducible elements, G.C.D., Maximal and prime ideals, Euclidean domains, Principal ideal domains, Unique factorization domains, Examples and counterexamples, Polynomial rings over domains, Eisenstein's irreducibility criterion,

**Books Recommended:**


1. N. Herstein, Topics in Algebra, Wiley Eastern, 1975.
2. P. B. Bhattacharya, S. K. Jain and S. R. Nagpal, Basic Abstract Algebra ($2^{nd}$ Edition), Cambridge University Press, Indian Edition 1977.
3. Ramji Lal, Algebra 1 and Algebra 2, Infosys Science foundation Series in Mathematical Sciences, Springer, Singapore, 2017.

4. D. S. Dummit and R.M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
5. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
6. J. B. Fraleigh, A first course in Abstract Algebra, Pearson Education, inc. 2002.


## Paper II: MAT 102 (Point-Set Topology)

**Unit I**

Countable and uncountable sets, Infinite sets and the axiom of choice, Cardinal numbers and its arithmetic, Schroeder-Bernstein theorem, Zorn's Lemma, Well ordering principle.

**Unit II**

Topological spaces, Closed sets, Open sets, Closure, Dense subsets, Neighbourhoods, exterior of a set, interior of a set, closure of a set, boundary of a set, Accumulation points and derived sets, Bases and subbases, Subspaces and relative topology.

**Unit III**

Separable space, Neighbourhood systems, first countable space, second countable space, Continuous functions and its characterizations via the closure and interior, open map, closed map, Homeomorphism, product of two spaces, quotient of a space, Compact space, Connected space, path connected space, components.

**Unit IV**

Separation axioms $T1$-space, $T2$ -space, regular space, $T3$ -space, completely regular space, normal space, $T4$ - space, their characterizations and basic properties.

**Unit V**

Nets and filters, Topology and convergence of nets. Hausdorffness and nets, Filters and their convergence, Ultra filters, Canonical way of converting nets to filters and vice-versa, Tychonoff Theorem (Statements and Sketch of proofs).

**Books Recommended:**


1. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
2. J. R. Munkres, Topology, Narosa Publishing House, New Delhi, 2005.
3. K. D. Joshi, Introduction to General Topology, Wiley Eastern, 1983
4. S.W. Davis Topology, Tata McGraw Hill, 2006
5. Sze-Tsen Hu, Elements of General Topology, Holden-Day Inc., 1964.

**Further Reading:**

1. N. Bourbaki, General Topology, Part I, Addison-Wesley, 1966.
2. J. Dugundji, Topology, Prentice-Hall of India, 1966.

## Paper III: MAT 103 (Complex Analysis)

**Unit I**

Algebra of complex numbers, Extended complex plane and stereographic projection, complex differentiability, Cauchy-Riemann equations, analytic functions, harmonic functions, harmonic conjugates, analyticity of functions defined by power series, the exponential, trigonometric, hyperbolic functions and their properties.

**Unit II**

Logarithmic functions, Branch of logarithm, power of a complex number, line integral, basic properties of contour integration, M-L inequality, fundamental theorem of contour integration, Cauchy's integral theorem, Cauchy-Goursat theorem (statement only), Cauchy's integral formula, Cauchy's integral formula for higher derivatives, Morera's theorem, Riemann's Removability theorem.

**Unit III**

Taylor's theorem, zeros of an analytic function, Laurent's theorem, the identity/uniqueness theorem for analytic functions, the identity theorem for power series, Maximum modulus theorem, Schwarz' lemma and its consequences, Cauchy's estimate, Liouville's theorem, A Generalized version of Liouville's theorem, the fundamental theorem of algebra.

**Unit IV**

Conformal mappings, Möbius transformations and its properties, the group of Möbius transformations, circles and lines under Möbius maps, cross ratio and its invariance property, Principle of symmetry (statement only), Conformal self maps of disks and half planes.

**Unit V**

Singularities of functions, Non-isolated singularities: removable singularity, poles and essential singularities, Characterization of removable singularity, pole and essential singularity, Characterizing singularities via Laurent series expansion, isolated singularities at $\infty$, meromorphic functions, Picard's theorem, Casoratti-Weierstrass theorem, residues, Cauchy's residue theorem, evaluation of definite and improper integrals using contour integration, argument principle, Rouche's theorem, open mapping theorem (statement only).

**Books Recommended:**


1. S. Ponnusamy and H. Silverman, Complex Variables, Birkhäuser, Inc., Boston, MA, 2006.
2. J. B. Conway, Functions of One Complex Variable, Narosa Publishing House, New Delhi, 2002.
3. V. Ahlfors, Complex Analysis (Third Edition), McGraw-Hill, 1979.
4. S.Ponnusamy, Foundation of complex analysis, Narosa publication, 2003.


**Paper IV: MAT 104 (Elementary Differential Geometry)**

**Unit I**

Curves in space $R^3$, Parameterization of curves, Regular curves, Helices, Arc length, reparameterization (by arc length), tangent, principal normal, binormal, osculating plane, normal plane, rectifying plane, curvature and torsion of smooth curves, Frenet- Serret formulae, Frenet approximation of a space curve.

**Unit II**

Osculating circle, osculating sphere, spherical indicatrices, involutes and evolutes, intrinsic equations of space curves, isometries of $R^3$, fundamental theorem of space curves, surfaces in $R^3$, regular surfaces, co-ordinate neighborhoods, parameterized surfaces, change of parameters, level sets of smooth functions on $R^3$, surfaces of revolution, tangent vectors, tangent plane, differential of a map.

**Unit III**

Normal fields and orientability of surfaces, angle between two intersecting curves on a surface, Gauss map and its properties, Weingarten map, second and third fundamental forms, classification of points on a surface.

**Unit IV**

Curvature of curves on surfaces, normal curvature, Meusnier theorem, principal curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, lines of curvature, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae for the Gaussian curvature, isometries of surfaces, Gauss Theorem Egregium (statement only).

**Unit V**

Christoffel symbols, Gauss formulae, Weingarten formulae, Gauss equations, Codazzi-Mainardi equations, curvature tensor, geodesics, geodesics on a surface of revolution, geodesic curvature of a curve, Gauss-Bonnet Theorem (statement only).

**Books Recommended:**


1. D. Somasundaram, Differential Geometry, A First Course, Narosa Publishing House, New Delhi, 2005.
2. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.
3. J. A. Thorpe, Elementary Topics in Differential Geometry, Springer (Undergraduate Texts in Mathematics), 1979.
4. B. O' Neill, Elementary Differential Geometry, Academic Press, 1997.
5. A. Pressley, Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.


## Paper V: MAT 105 (Classical Mechanics)

### Unit I

The momentum of a system of particles, the linear and the angular momentum, rate of change of momentum and the equations of motion for a system of particles, principles of linear and angular momentum, motion of the center of mass of a system, theorems on the rate of change of angular momentum about different points, with special reference to the center of mass, the kinetic energy of a system of particles in terms of the motion relative to the center of mass of the system.

Rigid bodies as systems of particles, general displacement of a rigid body, the displacement of a rigid body about one of its points and the concept of angular velocity, computation of the angular velocity of a rigid body in terms of the velocities of two particles of the system chosen appropriately.

### Unit II

The angular momentum and kinetic energy of a rigid body in terms of inertia constants, Equations of motion. Euler's dynamical equations of motion, Euler's geometrical equations of motion, Motion under no forces, the invariable line and the invariable cone, Instantaneous axis of rotation, Eulerian angles

### Unit III

Generalized co-ordinates, geometrical equations, holonomic and non-holonomic systems, configuration Space, Lagrange's equations using D' Alembert's Principle for a holonomic conservative system, Lagrangian function, deduction of equation of energy when the geometrical equations do not contain time $t$ explicitly, Lagrange's multipliers case, deduction of Euler's dynamical equations from Lagrange's equations, Lagrange equations for impulsive motion.

### Unit IV

Generalized momentum and the Hamiltonian for a dynamical system, Hamilton's canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, phase space and Hamilton's Variational principle, Hamilton's principle function, the principle of least action, canonical transformations, conditions of canonicality, Hamilton-Jacobi (H-J) equation of motion (outline only), Poisson-Brackets, Poisson-Jacobi identity, Poisson's first theorem.

**Unit V**

Theory of small oscillations, Lagrange's method, normal (principal) co-ordinates and the normal modes of oscillation, small oscillations under holonomic constraints, stationary property of normal modes, orthogonality of normal modes.

**Books Recommended:**


1. S.L. Loney, Dynamics of Rigid Bodies, CBS Publishers, New Delhi, 1913.
2. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Company, London, 1969.
3. N.C. Rana and P.S. Joag, "Classical Mechanics", Tata McGraw Hill, 1991.
4. E. A. Milne, Vectorial Mechanics, Methuen & Co. Ltd., London, 1965.
5. L. A. Pars, A Treatise on Analytical Dynamics, Heinemann, London, 1968.
6. N. Kumar, Generalized Motion of Rigid Body, Narosa Publishing House, New Delhi, 2004.
7. A. S. Ramsey, Dynamics, Part II, CBS Publishers & Distributors, Delhi, 1985.

# M.Sc. (Mathematics) Semester-II

## Paper I: MAT 201 (Module Theory)

**Unit I**

Modules, Submodules, Faithful modules, Factor modules, Exact sequences, Five lemma, Products, co-products and their universal property, Direct summands, Annihilators, Free modules, External and internal direct sums.

**Unit II**

Homomorphism extension property, Equivalent characterization as a direct sum of copies of the underlying ring, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness.

**Unit III**

Projective modules, Injective modules, Baer's characterization, Divisible groups, Noetherian modules and rings, Equivalent characterizations, Submodules and factors of noetherian modules, Hilbert basis theorem (statement only).

**Unit IV**

Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into torsion and a torsion free submodule, $p$-primary components, Decomposition of $p$-primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Structure of finite abelian groups.

**Unit V**

Similarity of matrices and $F[x]$-module structure, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Diagonalizable and nilpotent parts of a linear transformation, Jordan-Chevalley Theorem.

**Books Recommended:**


1. P. Ribenboim, Rings and Modules, Wiley Interscience, New York, 1969.
2. J. Lambek, Lectures on Rings and Modules, Blaisedell, Waltham, 1966.
3. Ramji Lal, Algebra 2, Infosys Science foundation Series in Mathematical Sciences, Springer, Singapore, 2017.
4. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
5. N. S. Gopalkrishnan, University Algebra, Wiley Eastern Ltd., New Delhi, 1986.

**Paper II: MAT 202 (Integral Equations and Partial Differential Equations)**

**Unit I**

Linear integral equations-Definition and classification of conditions, Special kinds of Kernels, Eigen values and eigen functions, Solution of linear integral equations with separable Kernels.

**Unit II**

Fredholm alternative, Fredholm Theorem, Fredholm alternative theorem, Approximate method, Method of successive approximations- Iterative scheme, Solution of Fredholm and Volterra integral equation, Results about resolvent Kernel, Relation between differential and integral equations.

**Unit III**

Partial differential equations (P.D.E.'s), First order P.D.E.'s, Classification of first order P.D.E.'s, Complete, general and singular integrals, Lagrange's or quasi-linear equations, Integral surfaces through a given curve, Charpit's Method, Orthogonal surfaces to a given system of surfaces, Characteristic curves.

**Unit IV**

Classification of second order P.D.E.'s, Reduction to canonical forms, Linear partial differential equations with constant coefficients,

**Unit V**

Method of separation of variables: Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, Boundary value problems for transverse vibrations in a string of finite length and heat diffusion in a finite rod, D'Alembert's solution of the one dimentional wave equation.

**Books Recommended:**


1. I. N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill, 1957.
2. T. Amaranath, An Elementary Course in Partial Differential Equations, Narosa Publishing House, New Delhi, 2005.
3. R. P. Kanwal, Linear Integral Equations, Birkhäuser, Inc., Boston, MA, 1997.
4. Tyn Myint-U, Partial Differntial Equations of Mathematical Physics, Elseveir Publications.
5. Erwin Kreyszig, Advanced Engineering Mathematics, $8^{th}$ Edition, Wiley Student Publications, India, 2010.

## Paper III: MAT 203 (Mathematical Methods)

**Unit I**

Periodic functions, Trigonometric series, Fourier series, Euler formulas, Functions having arbitrary periods, Even and Odd functions, Half-range expansions, Determination of Fourier coefficients without integration, Approximation by trigonometric polynomials, Square error.

**Unit II**

Orthogonal and Orthonormal sets of functions, Generalized Fourier series, Sturm-Liouville problems, Examples of Boundary-value problems which are not Sturm-Liouville problems, Green's functions.

**Unit III**

Laplace Transform, Properties of Laplace transform, Laplace Transform of a periodic function, error function and Dirac Delta function, Inverse transform Shifting and linearty property of Laplace transform, Laplace Transform of the derivatives and of the Integrals of a function, Derivatives and Integrals of Transforms, Convolution theorem (Faltung theorem).

**Unit IV**

Fourier Transform, Properties of Fourier transform (linearity property, change of scale, shifting property, modulation property), Fourier Integrals, Fourier Cosine and Sine Integrals, Inverse Fourier Transform, Fourier Cosine and Sine Transform, Complex form of the Fourier Transform, Convolution theorem.

**Unit V**

Finite Fourier transform, Finite sine transform, Finite Cosine transform, Solution of ordinary differential equations using Laplace transform method, Solution of wave equation using the method of Fourier transform: D'Alembert's solution of the wave equation.

**Books Recommended:**


1. E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8$^{th}$ Edition, 2001.
2. J. H. Davis, Methods of Applied Mathematics with a MATLAB Overview, Birkhäuser, Inc., Boston, MA, 2004.

## Paper IV: MAT 204 (Measure and Integration)

**Unit I**

Cardinality of a set, Arithmatic of cardinal numbers, Schröder-Bernstein theorem, The Cantor's ternary set and its properties, The Cantor-Lebesgue function.

**Unit II**

Semi-algebras, algebras, monotone class, σ-algebras, measure and outer measures, Caratheödory extension process of extending a measure on semi-algebra to generated σ -algebra, completion of a measure space.

**Unit III**

Borel sets, Lebesgue outer measure and Lebesgue measure on *R*, translation invariance of Lebesgue measure, existence of a non-measurable set, characterizations of Lebesgue measurable sets.

**Unit IV**

Measurable functions, Characterization of measurable functions, Linearity and products of measurable functions, Borel and Lebesgue measurable functions, Characteristic functions, simple functions and their integrals, Lebesgue integral on *R* and its properties, Characterizations of Riemann and Lebesgue integrability.

**Unit V**

Littlewood's three principles (statement only), Bounded convergence theorem, Lebesgue monotone convergence theorem, Fatou's lemma, Lebesgue dominated convergence theorem.

**Books Recommended:**


1. K. Rana, An Introduction to Measure and Integration, Second Edition, Narosa Publishing House, New Delhi, 2005.
2. P. R. Halmos, Measure Theory, Grand Text Mathematics, 14, Springer, 1994.
3. E. Hewit and K. Stromberg, Real and Abstract Analysis, Springer, 1975.
4. K. R. Parthasarathy, Introduction to Probability and Measure, TRIM 33, Hindustan Book Agency, New Delhi, 2005.
5. H. L. Royden and P. M. Fitzpatrick, Real Analysis, Fourth edition, Prentice Hall of India, 2010.

**Paper V: MAT 205 (Discrete Mathematics)**

**Unit I**

Mathematical Logic, Statement calculus: Propositional logic, Logic operators or connectives, Well formed formula (wff), Construction of truth-table for a formula, Equivalence of formulas, Tautology, Contradiction argument, Valid argument, Proving validity by truth-table methods, Inference theory of statement calculus, Minimal sets of logic operators. Predicate calculus: Statement function and statement, Proving validity by the deduction method, Inference rules, Proving validity by the method of contradiction.

**Unit II**

Lattice theory and Boolean algebra Lattice Theory: partial order relation, Partially ordered set, Totally ordered set, Hasse Diagrams, Lattice, Lattice as an algebraic system, Bounded lattice, Complemented lattice, Distributive lattice, Direct product, Lattice homomorphism.

**Unit III**

Boolean algebra: Boolean functions, Principle of duality, Boolean function minimization, Sum of products and product of sums form, Normal forms, Conversion of normal forms into principal normal forms, Boolean function minimization, Logic circuits, Designing logic circuits.

**Unit IV**

Automata theory, Finite state automaton, Types of automaton, Deterministic finite state automaton, Non deterministic finite state automaton, Non deterministic finite-state automaton with ε, Equivalence of NFA and DFA, Equivalence of NFA and NFA-ε, Equivalence of NFA-ε and DFA, Finite state Machines: Moore and Mealy machine, and their conversion, Turing machine.

**Unit IV**

Grammars and Languages, Regular language, Regular expression Equivalence of Regular language and finite state automaton, Grammar: Context-free and Context-sensitivemgrammar, LR Grammar: Contruction of LR(0) parsing table, Construction of LR(1) parsing table, Decision algorithms for CFL.

**Books Recommended:**


1. John E. Hoprcroft, Rajeev Motwani, Jeffrey D. Ullman: Introduction to Aotomata Theory, Languages and Computation, Pearson Education, 2000.
2. Mendelson, Elliott: Introduction to Mathematical Logic, Chapman & Hall, 1997.
3. Arnold B. H.: Logic and Boolean Algebra, Prentice Hall, 1962.
4. K. H. Rosen: Discrete Mathematics and its applicatons, MGH 1999.

## M.Sc. (Mathematics) Semester-III

### Paper I: MAT 301 (Functional Analysis)

**Unit I**

Norm and its properties, Normed linear spaces, Banach spaces, the sequence spaces and the function spaces as Banach spaces, Characterization of Continuous linear transformations between two normed spaces, Bounded linear operators, B(X,Y) as a normed linear space.

**Unit II**

Hahn-Banach Theorem, Open mapping theorem, Closed graph theorem, Banach-Steinhaus theorem (only statement), Uniform boundedness principle (only statement).

**Unit III**

Conjugate spaces, Weak and Weak*-topology on a conjugate space, Simple Application to reflexive separable spaces and to the Calculus of Variation.

**Unit IV**

Hilbert Spaces, Schwarz's inequality, orthogonal complement of a subspace, orthonormal bases, Continuous linear functionals on Hilbert spaces, Riesz Representation Theorem, Reflexivity of Hilbert Spaces, Applications of polarization identity.

**Unit V**

The adjoint of an operator, Self adjoint operators, Normal and unitary operators, Projections. Finite dimensional spectral theory – Spectrum of an operator, the Spectral theorem.

**Books Recommended:**


1. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
2. S. Ponnusamy, Foundations of Functional Analysis, Narosa Publishing House, New Delhi, 2002.
3. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1966.
4. A. E. Taylor, Introduction to Functional Analysis, John Wiley, 1958.
5. N. Dunford and J. T. Schwartz, Linear Operators, Part-I, Interscience, 1958.
6. R. E. Edwards, Functional Analysis, Holt Rinehart and Winston, 1965.
7. C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice- Hall of India, 1987.

**Paper II: MAT 302 (Theory of Ordinary Differential Equations)**

**Unit I**

Initial and Boundary Value Problems, Picard's method of successive approximations, Lipschitz conditions, Sufficient conditions for being Lipschitzian in terms of partial derivatives, Examples of Lipschitzian and Non-Lipschitzian functions, Existence and Uniqueness theorem for first order initial value problem, Differential equations of first order not solvable for the derivative.

**Unit II**

Uniqueness of solutions with a given slope, Singular solutions, *p*- and *c*-discriminant equations of a differential equation and its family of solutions respectively, Envelopes of one parameter family of curves, singular solutions as envelopes of families of solution curves, Sufficient conditions for existence and non-existence of singular solutions, examples.

**Unit III**

Systems of I order equations arising out of equations of higher order, Norm of Euclidean spaces convenient for analysis of systems of equations, Lipschitz condition for functionsfrom $R^{n+1}$ to $R^n$, Local existence and uniqueness theorems for systems of I order equations, Gronwall's inequality, Global existence and uniqueness theorems for existence of unique solutions over whole of the given interval and over whole of $R$, Existence theory for equations of higher order, Conditions for transformability of a system of I order equations into an equation of higher order.

**Unit IV**

Linear independence and Wronskians, General solutions covering all solutions for homogeneous and non-homogeneous linear systems, Abel's formula, Method of variation of parameters for particular solutions, Linear systems with constant coefficients, Matrix methods, Different cases involving diagonalizable and non-diagonalizable coefficient matrices, Real solutions of systems with complex eigenvalues.

**Unit V**

Ordinary and singular points, Power series solutions, Frobenius' generalized power series method, Indicial equation, different cases involving roots of the indicial equation, Regular and logarithmic solutions near regular singular points

Legendre's equation, Solution by power series method, polynomial solution, Legendre polynomial, Rodrigues' formula, Generating function, Recurrence relations, Bessel's equation, Bessel functions of I and II kind, Recurrence relations, Bessel functions of half-integral orders, Zeros of Bessel functions, Orthogonality relations, Generating function.

**Books Recommended:**

1. B. Rai, D. P. Choudhury and H. I. Freedman, A Course in Ordinary Differential Equations, Narosa Publishing House, New Delhi, 2002.
2. L. Collatz, The Numerical Treatment of differential equations, Springer-Verlag, 1960.
3. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1968.
4. D.V. Widder, Advanced Calculus, Prentice Hall, 1961.

## Paper III: MAT 303 (Galois Theory)

### Unit I

Field extensions, Degree of extension, Finite extensions, Algebraic and transcendental elements, Algebraic and transcendental extensions, Simple extensions, Primitive element of the extension.

### Unit II

Splitting fields and their uniqueness, Normal extensions, Separable extensions, Perfect fields, Transitivity of separability, Algebraically closed field and Algebraic Closure, Normal closures, Dedekind's theorem.

### Unit III

Automorphisms of fields, K-automorphisms, Fixed fields, Galois group of the extension field, Abelian extension, Cyclic extension, Galois extensions, Fundamental theorem of Galois theory, Computation of Galois groups of polynomials.

### Unit IV

Finite fields, Existence and uniqueness, Subfields of finite fields, Characterization of cyclic Galois groups of finite extensions of finite fields, Solvability by radicals, Galois' characterization of such solvability, Generic polynomials, Abel-Ruffini theorem, Geometrical constructions.

### Unit V

Cyclotomic extensions, Cyclotomic polynomials and its computations, Cyclotomic extensions of $Q$, Galois groups of splitting fields of $x^n - 1$ over $Q$.

### Books Recommended:

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. I. A. Adamson, An Introduction to Field Theory. Oliver & Boyd, Edinburgh, 1964.
3. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
4. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Ltd., New Delhi, 1986.
5. F. W. Anderson and K. R. Fuller, Rings and Categories of Modules, Springer, New York, 1974.

## Paper IV: MAT 304 (Differentiable Manifolds)

**Unit I**

Topological manifolds, compatible charts, smooth manifolds, examples, smooth maps and diffeomorphisms, definition of a Lie group, examples.

**Unit II**

Tangent and cotangent spaces to a manifold, Derivative of a smooth map, Immersions and submersions, Submanifolds, vector fields, algebra of vector fields, $\varphi$-related vector fields, left and right invariant vector fields on Lie groups.

**Unit III**

Integral curves of smooth vector fields, complete vector fields, flow of a vector field, Distributions, n-dimensional real vector space, Contravariant vectors, Dual vector space, Covariant vectors.

**Unit IV**

Tensor product of vector spaces, Second order tensors, Tensors of type (r, s), Symmetry and skew symmetry of tensors, Fundamental algebraic operations, Quotient law of tensors, Tensor fields on manifolds, Differential forms, Exterior product, Exterior differentiation, Pull-back differential forms.

**Unit V**

Affine connections (covariant differentiation) on a smooth manifold, torsion and curvature tensors of an affine connection, Identities satisfied by curvature tensor.

**Books Recommended:**


1. S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Vol. 1, Interscience Publishers, 1963.
2. T. J. Willmore, Riemannian Geometry, Oxford Science Publication, 1993.
3. S. Kumaresan, A Course in Differential Geometry and Lie Groups, Hindustan Book Agency, New Delhi, 2002.
4. M. Spivak, A Comprehensive Introduction to Differential Geometry, Vols. 1-5, Publish or Perish, Inc., Houston, 1999.
5. W. M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, revised, 2003.

## Paper V: MAT 305 (Fluid Mechanics)

**Unit I**

Body forces and surface forces, Nature of stresses, Transformation of stress components, Stress invariants, Principal stresses, Nature of strains, Rates of strain components, Relation between stress and rate of strain components, General displacement of a fluid element, Newton's law of viscosity, Navier-Stokes equation (sketch of proof).

**Unit II**

Equation of motion for inviscid fluid, Energy equation, Helmholtz's vorticity theorem and vorticity equation, Kelvin's circulation Theorem, Mean Potential over a spherical surface, Kelvin's Minimum kinetic energy Theorem, Acyclic irrotational motion.

**Unit III**

Two dimensional irrotational motion – Complex potential, Concept of line vortices, Vortex rows and the Karman vortex street, Milne-Thomson Circle Theorem, Complex potential for a uniform flow past a circular cylinder, Streaming and circulation about a fixed circular cylinder, Blasius Theorem, Conformal transformation: Uniform line distributions (source, vortex and doublet) under conformal transformation.

**Unit IV**

Three dimensional irrotational flow: Axisymmetric flow, Stokes stream function, Axisymmetric potential flow, Butler's sphere theorem and its applications, Liquid streaming past a stationary sphere, Uniform motion of a sphere in a liquid at rest at infinity, Concentric spheres (Problem of Initial motion).

**Unit V**

Gravity waves – Surface waves on the infinite free surface of liquids, Waves at the interface between finitely and infinitely deep liquids.

**Books Recommended:**


1. A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd. 1960.
2. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2nd Edition, 1987.
3. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. van Nostrand Comp. Ltd., London, 1968.
4. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
5. F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers, New Delhi, 2004.

## M.Sc. (Mathematics) Semester-IV

***(One core and three elective papers)***

### Paper I: MAT 401 (Wavelets)

**Unit I**

The discrete Fourier transform and the inverse discrete Fourier transform, their basic properties and computations, the fast Fourier transform, The translation invariant linear transformation.

**Unit II**

Construction of first stage wavelets on $\mathbb{Z}_N$, Shannon wavelets, Daubechies' D6 wavelets on $\mathbb{Z}_N$. Description of $l^2(\mathbb{Z})$, $L^2[-\pi, \pi)$, $L^2(\mathbb{R})$, their orthonormal bases, Fourier transform and convolution on $l^2(\mathbb{Z})$, wavelets on $\mathbb{Z}$, Haar wavelets on $\mathbb{Z}$, Daubechies' D6 wavelets for $l^2(\mathbb{Z})$.

**Unit III**

Orthonormal bases generated by a single function in $L^2(\mathbb{R})$, Fourier transform and inverse Fourier transform of a function $f$ in $L^1(\mathbb{R}) \cap L^2(\mathbb{R})$, Parseval's relation, Plancherel's formula, Orthonormal wavelets in $L^2(\mathbb{R})$, Balian-Low theorem.

**UNIT IV**

Multi-resolution analysis and MRA wavelets, Low pass filter, Characterizations in multiresolution analysis, compactly supported wavelets, band-limited wavelets.

**UNIT V**

Franklin wavelets on $\mathbb{R}$, Dimension function, Characterization of MRA wavelets (Sketch of the proof), Minimally Supported Wavelets, Wavelet Sets, Characterization of two-interval wavelet sets, Shannon wavelet, Journe's wavelet, Decomposition and reconstruction algorithms of Wavelets.

**Books Recommended:**

1. Eugenio Hernández and Guido Weiss, A First Course on Wavelets, CRC Press, 1996.
2. Ingrid Daubechies, Ten Lectures on Wavelets, CBS-NFS Regional Conferences in Applied Mathematics, 61, SIAM, 1992.
3. Michael W. Frazier, An Introduction to Wavelets through Linear Algebra, Springer-Verlag, 1999.
4. C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.

## Paper II: MAT 402 (Linear Programming)

### Unit I

Linear Programming and examples, Convex Sets, Hyperplane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Determination of Optimal solutions.

### Unit II

Extreme Point & graphical methods, Simplex method, Charnes-M method, Two phase method, unrestricted variables, Duality theory, Dual linear Programming Problems, fundamental properties of dual Problems.

### Unit III

Complementary slackness, unbounded solution in Primal. Dual Simplex Algorithm, Sensitivity analysis, Parametric Programming, Revised Simplex method.

### Unit IV

Transportation Problems, Balanced and unbalanced Transportation problems, U-V method, Paradox in Transportation problem, Assignment problems.

### Unit V

Integer Programming problems: Pure and Mixed Integer Programming problems, 0-1 programming problem, Gomary’s algorithm, Branch & Bound Technique. Travelling salesman problem.

### Books Recommended:

1. Suresh Chandra, Jayadeva and Aparna Mehra: Numerical Optimization with Applications, Narosa Publishing House, 1st Edition, 2009.
2. G. Hadley: Linear Programming, Narosa Publishing House, 6th edition. 1995.
3. S. M. Sinha: Mathematical Programming,Theory and Methods, 1st Edition, Elsevier, 2006.
4. N. S. Kambo: Mathematical Programming Techniques, 1984, Affiliated East-West Press Pvt. Ltd. New Delhi, Madras (Reprint 2005) revised Edition.

***OR***

**Paper II: MAT 403 (Riemannian Geometry)**

**Unit I**

Riemannian metrics, Riemannian manifolds, examples, Affine connections, Covariant differentiation of tensor fields, Covariant derivative along a curve, Parallel transport, Levi-Civita connection, Fundamental theorem of Riemannian geometry.

**Unit II**

Differential operators on Riemannian manifolds, Gradient vector fields, Divergence of a vector field, Laplacian operator, Lie derivative of a tensor field with respect to a vector field.

**Unit III**

Riemannian curvature tensor, Identities satisfied by Riemannian curvature tensor, Sectional curvature, Schur's Theorem, Ricci curvature, Scalar curvature, Einstein manifolds, Isometries, Notion of covering spaces, Pull-back metrics via diffiomorphisms.

**Unit IV**

Length of a curve, Riemannian distance function, Geodesics, Local existence and uniqueness for geodesics, Exponential map, Gauss lemma, Minimizing properties of geodesics, Geodesic normal coordinates.

**Unit V**

Jacobi fields, Conjugate points, Complete Riemannian manifolds, Hopf-Rinow Theorem, The Theorem of Hadamard, Riemannian submanifolds, Second fundamental form, Gauss equation, Ricci equation, Model spaces of constant curvature.

**Books Recommended:**

1. M. P. do Carmo, Riemannian Geometry, Birkhauser, 1992.
2. P. Peterson, Riemannian Geometry, Springer, 2006.
3. J. Jost, Riemannian Geometry and Geometric Analysis, Springer, $6^{th}$ edition, 2011.
4. J. M. Lee, Riemannian Manifolds: An Introduction to Curvature, Springer, 1997.
5. S. Gallot, D. Hullin, J. Lafontaine, Riemannian Geometry, Springer, $3^{rd}$ edition, 2004.

**<u>Paper III: MAT 404 (Algebraic Number Theory)</u>**

**Unit I**

Archimedean and non-Archimedean absolute values, approximation theorem, absolute values on $Q$ and $F[x]$, completion of a field, Ostrowski's theorem.

**Unit II**

$p$-adic numbers and $p$-adic integers, arithmetic in $Qp$ , Hensel's lemma, sequence and series in $Qp$ , exponential and logarithmic series in $Qp$.

**Unit III**

Number fields, the ring of algebraic integers, calculation for quadratic, cubic and cyclotomic case, norms and traces, discriminants.

**Unit IV**

Dedekind domains, unique factorization of ideals, splitting of primes in extensions, decomposition and inertia group, unramified and ramified extensions, the Frobenius automorphism.

**Unit V**

The ideal class group, lattices in $Rn$ , finiteness of the class number, the Dirichlet unit theorem.

**Books Recommended:**


1. J. S. Milne, Algebraic Number Theory, 2011.
2. N. Jacobson, Basic Algebra, Vol. 2, Hindustan Publishing Corporation, New Delhi, 1994.
3. D. A. Marcus, Number Fields, Springer-Verlag, 1977.
4. J. Esmonde and M. Ram Murty, Problems in Algebraic Number Theory, Graduate Text in Mathematics, 190, Springer-Verlag, 1999.
5. A. M. Roberts, A Course in *p*-adic Analysis, Graduate Text in Mathematics, 198, Springer-Verlag, 2000.

***OR***

**Paper III: MAT 405 (Nonlinear Analysis)**

**Unit I**

Compactness in Metric spaces, Measure of Noncompactness, Normed spaces, Banach spaces, Hilbert spaces, Uniformly convex, strictly convex and reflexive Banach spaces, Lipschitzian and contraction mapping, Banach's contraction principle, Application to Volterra and Fredholm integral equations.

**Unit II**

Nonexpansive, asymptotically nonexpansive, accretive and quasinonexpansive mappings, Fixed point theorems for nonexpansive mappings, Nonexpansive operators in Banach spaces satisfying Opial's conditions, The demiclosedness principle.

**Unit III**

Schauder's fixed point theorem. Condensing maps. Fixed points for condensing maps, The modulus of convexity and normal structure, radial retraction, Sadovskii's fixed point theorem, Set-valued mappings.

**Unit IV**

Fixed point iteration procedures, The Mann Iteration, Lipschitzian and Pseudocontractive operators in Hilbert spaces, Strongly pseudocontractive operators in Banach spaces, The Ishikawa iteration, Stability of fixed point iteration procedures.

**Unit V**

Iterative solution of Nonlinear operator equations in arbitrary and smooth Banach spaces, Nonlinear *m*-accretive operator, Equations in reflexive Banach spaces.

**Books Recommended:**

1. V. Berinde, Iterative Approximation of Fixed Points, Lecture Notes in Mathematics, No. 1912, Springer, 2007.
2. M. A. Khamsi and W. A. Kirk, An Introduction to Metric Spaces and Fixed Point Theory, John Wiley & Sons, New York, 2001.
3. Sankatha P. Singh, B. Watson and P. Srivastava, Fixed Point Theory and Best Approximation: The KKM-map Principle, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997.
4. V. I. Istratescu, Fixed Point Theory, An Introduction, D. Reidel Publishing Co., 1981.
5. K. Goebel and W. A. Kirk, Topic in Metric Fixed Point Theory, Cambridge University Press, 1990.

## Paper IV: MAT 406 (Advanced Fluid Mechanics)

**Unit I**

Stress Principle of Cauchy, Equations for conservation of linear and angular Momentum, Constitutive equations for Newtonian fluids, Navier-Stokes equations in vector and tensor forms, Navier-Stokes equations in orthogonal coordinate systems (particularly in Cartesian, cylindrical and spherical coordinate systems).

**Unit II**

Vorticity equations, Energy dissipation due to viscosity, Dynamical similarity and dimensionless numbers and their significance in fluid dynamics, Some Exact solutions – Fully developed plane Poiseuille and Couette flows between parallel plates, Steady flow between pipes of uniform cross-section.

**Unit III**

Couette flow between coaxial rotating cylinders, Small Reynolds number flow – Flow between steadily rotating spheres, Stokes equations, Dynamic equation satisfied by stream function, Relation between pressure and stream function, General stream function, solution of Stokes equations in spherical polar coordinates, Steady flow past a sphere, Drag on a body.

**Unit IV**

Flow past a circular cylinder, Stokes paradox, Oseen's equations, Elementary ideas about perturbation and cell methods as applied to slow flow problems, Boundary layer concept.

**Unit V**

Two dimensional boundary layer equations, Separation phenomena, method, Boundary layer on a semi-infinite plane, Blasius equation and solution, Karman's integral method, Displacement thickness, Momentum thickness and Energy thicknesses.

**Books Recommended:**

1. Z. U. A. Warsi, Fluid Dynamics, CRC Press, 2005.
2. J. Happel and H. Brenner, Low Reynolds Number Hydrodynamics, Kluwer Academic Publishers Group, Dordrecht, The Netherlands, 1983.
3. W. E. Langlois, Slow Viscous flow, Macmillan, 1964.
4. T. C. Papanastasiou, G. C. Georgiou and A. N. Alexandrou, Viscous Fluid Flow, CRC Press, 2000.
5. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. Van Nostrand Comp. Ltd. London, 1964.

***OR***

**Paper IV: MAT 407 (Representation Theory of Finite Groups)**

**Unit I**

Irreducible and completely reducible modules, Schur's Lemma, Jacobson density Theorem, Wedderburn Structure theorem for semisimple modules and rings. Group Algebra Maschke's Theorem.

**Unit II**

Representations of a group on a vector space, matrix representation of a group, equivalent and non-quivalent representations, Decomposition of regular representation, Number of irreducible representations.

**Unit III**

Characters, irreducible characters, Orthogonality relations, Integrality properties of characters, character ring, Burnside's *paqb* - Theorem.

**Unit IV**

Representations of direct product of two groups, Induced representations, The character of an induced representations, Frobenius reciprocity Theorem. Construction of irreducible representations of Dihedral group $D_n$ , Alternating group $A_4$ , Symmetric groups $S_4$ and $S_5$ .

**Unit V**

Mackey's irreducibility criterion, Clifford's Theorem, Statement of Brauer and Artin's Theorems.

**Books Recommended:**


1. M. Burrow, Representation Theory of Finite Groups, Academic Press, 1965.
2. L. Dornhoff, Group Representation Theory, Part A, Marcel Dekker, Inc., New York, 1971.
3. N. Jacobson, Basic Algebra II, Hindustan Publishing Corporation, New Delhi, 1983.
4. S. Lang, Algebra, 3rd ed., Springer, 2004.
5. J. P. Serre, Linear Representation of Groups, Springer-Verlag, 1977.


**Paper V: MAT 408 (Dissertation)**

**MAT 409 (Viva-Voce)**