

Kepler's laws of Planetary Motion

Johannes Kepler was a 16th century astronomer who established three laws which govern the motion of planets around the sun. These are known as Kepler's laws of planetary motion.

1. **The Law of Orbits:** All planets move in elliptical orbits, with the sun at one of the foci.
2. **The Law of Areas:** A line that connects a planet to the sun sweeps out equal areas in equal times.
3. **The Law of Periods:** The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

Kepler's laws were derived for orbits around the sun, but they apply to satellite orbits as well. The detailed description is given bellow,

1. Kepler's first law:

The planets move in elliptical orbits around the sun, with the sun at one of the two foci of the elliptical orbit. This means that the orbit or path of a planet around the sun is an ellipse i.e. an oval-shaped and not an exact circle. An elliptical path has two foci and the sun is at one of the two foci of the elliptical path. This law is important for us as it helps us discover if other stars have planets.

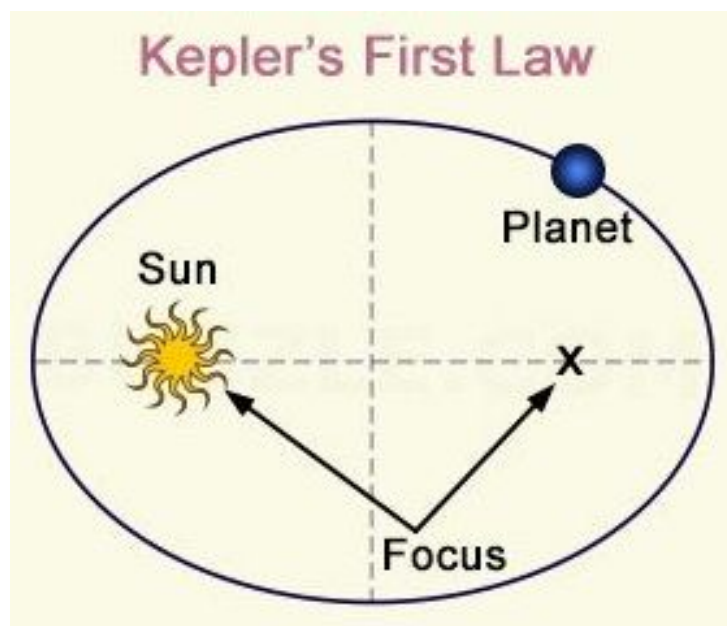


Figure-1

2. Kepler's second law:

Each planet revolves around the sun in such a way that the line joining the planet to the sun sweeps over equal areas in equal intervals of time. We know that a planet moves around the sun in an elliptical orbit with sun at one of its focus. Now, since the line joining the planet and the sun sweeps over equal areas in equal intervals of time, it means that a planet moves faster when it is closer to the sun and moves slowly when it is farther from the sun.

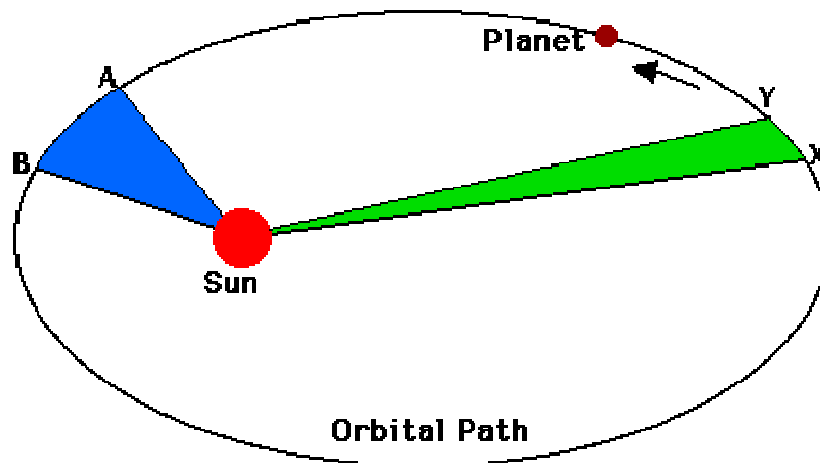


Figure-2

In the above figure a planet P is moving in an elliptical orbit around the sun. When the planet is nearer to the sun at position A, it travels faster and sweeps over an area ABC in time t . On the other hand, when the same planet is farther from the sun at position X, then it moves slowly but sweeps over an equal area XYZ in the same time t . ***Thus the Kepler's second law states that a planet does not move with constant speed around the sun. The speed is greater when the planet is nearer the sun and less when the planet is farther away from the sun.*** A planet could move around the sun with constant speed only if its orbit were a true circle and not an ellipse.

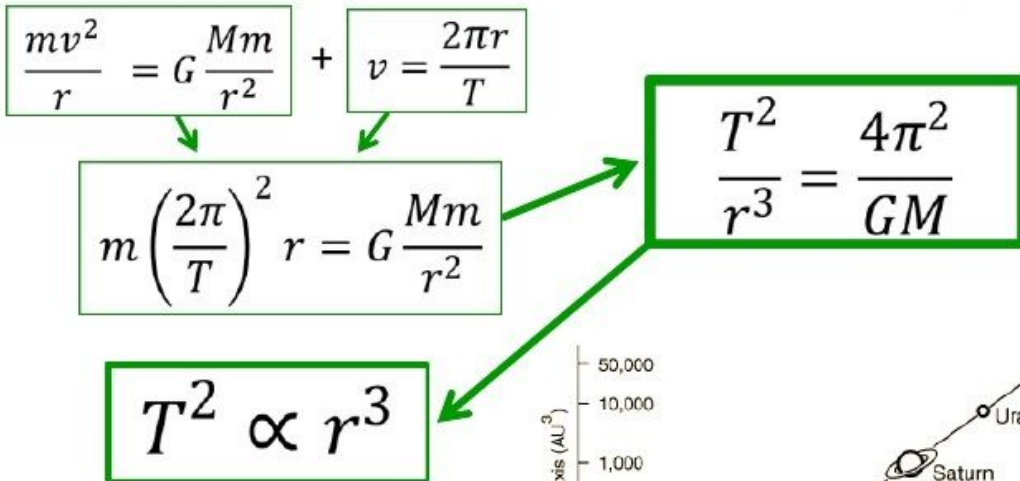
3. Kepler's third law:

The cube of the mean distance of a planet from the sun is directly proportional to the square of time it takes to move around the sun. With the help of Kepler's third law of planetary motion we can show how long does it takes to reach Mars, how long would it take for a spacecraft from earth to reach the Sun.

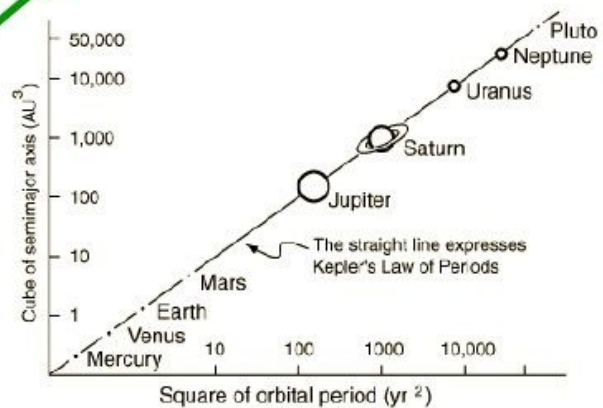
Though Kepler gave the laws of planetary motion but he could not give an explanation about the motion of planets. It was Newton who showed that the cause of motion of planets is the gravitational force which the sun exerts on them. In fact, Newton used the Kepler's third law of planetary motion to develop the law of universal gravitation

Kepler's 3rd Law

When something is in orbit, Centripetal Force is caused by Gravitational Force.



The 3rd Law: The **square of the orbital period** of a planet is **directly proportional** to the **cube of the semi-major axis** of its orbit



Bode's Law

In 1766 the German astronomer Titius observed an interesting regularity among the mean distances of planets known at the time: with two notable exceptions, they seemed to follow a progression, which (when converted to astronomical units) gave the mean distance as $0.4 + 0.3 \cdot 2^n$, with $n=0, 1, 2, 3...$ indicates the order of the planet from the Sun.

The Titius–Bode law (sometimes termed just Bode's law) is a hypothesis that the bodies in some orbital systems, including the Sun's, orbit at semi-major axes in a function of planetary sequence. The formula suggests that, extending outward, each planet would be approximately twice as far from the Sun as the one before. The hypothesis correctly anticipated the orbits of Ceres (in the asteroid belt) and Uranus, but failed as a predictor of Neptune's orbit and was

eventually superseded as a theory of Solar System formation. It is named after Johann Daniel Titius and Johann Elert Bode. Here is the tabulation, using Bode's law,

Planet	n	$0.4 + 0.3(2^n)$	Dist.(AU) from Sun
Mercury	---	0.4	0.387
Venus	0	0.7	0.723
Earth	1	1	1
Mars	2	1.6	1.524
???	3	2.8	???
Jupiter	4	5.2	5.203
Saturn	5	10	9.539

Lapse rate

The lapse rate is the rate at which an atmospheric variable, normally temperature in Earth's atmosphere, changes with altitude. Lapse rate arises from the word lapse, in the sense of a gradual change. It corresponds to the vertical component of the spatial gradient of temperature. Although this concept is most often applied to the Earth's troposphere, it can be extended to any gravitationally supported parcel of gas.

The decrease in the observed temperature with height is called the environmental or observed lapse rate. Traditionally, lapse rates are defined as the negative of the temperature gradient with respect to height, i.e.,

$$\Gamma = -dT/dz$$

Γ is the lapse rate given in units of temperature divided by units of altitude, T is temperature, and z is altitude.

The average lapse rate in the middle latitude troposphere is about $0.65^\circ\text{C}/100\text{ m}$. It is also known that the pressure decreases with height, so that a parcel of air that changes its altitude will undergo a pressure change, and therefore a temperature change as well. If one assumes that vertical displacements occur adiabatically, one can calculate the change in temperature associated with these displacements. This temperature change is described in terms of a process lapse rate. If the parcel moves vertically with no water vapor phase change, the process is called dry adiabatic, and the rate of cooling per unit rise is called the dry adiabatic lapse rate. Since we assume the process to be reversible, we can derive this process lapse rate from the reversible form of the first law of thermodynamics:

$$\Delta H = c_p dT - v dp$$

If adiabatic, $\Delta H = 0$ and: $c_p dT = v dp$ or $dT = v dp / c_p$

Adiabatic lapse rate: Change of temperature with a change in altitude of an air parcel without gaining or losing any heat to the environment surrounding the parcel. Types of Adiabatic lapse rate:

- 1. Dry adiabatic lapse rate:** Assumes a dry parcel of air. Air cools $1^\circ\text{C}/100\text{ m}$ rise in altitude ($5.4^\circ\text{F}/1000\text{ ft}$).

2. Wet adiabatic lapse rate: As parcel rises, H₂O condenses and gives off heat, and warms air around it. Parcel cools more slowly as it rises in altitude, $\approx 6^{\circ}\text{C}/1000\text{ m}$ ($\approx 3.5^{\circ}\text{F}/1000\text{ ft}$).

Ambient or prevailing lapse rate: The actual atmospheric temperature change with altitude, the actual lapse rate is modified by water content, wind, sunlight on the Earth's surface, and geographical features.

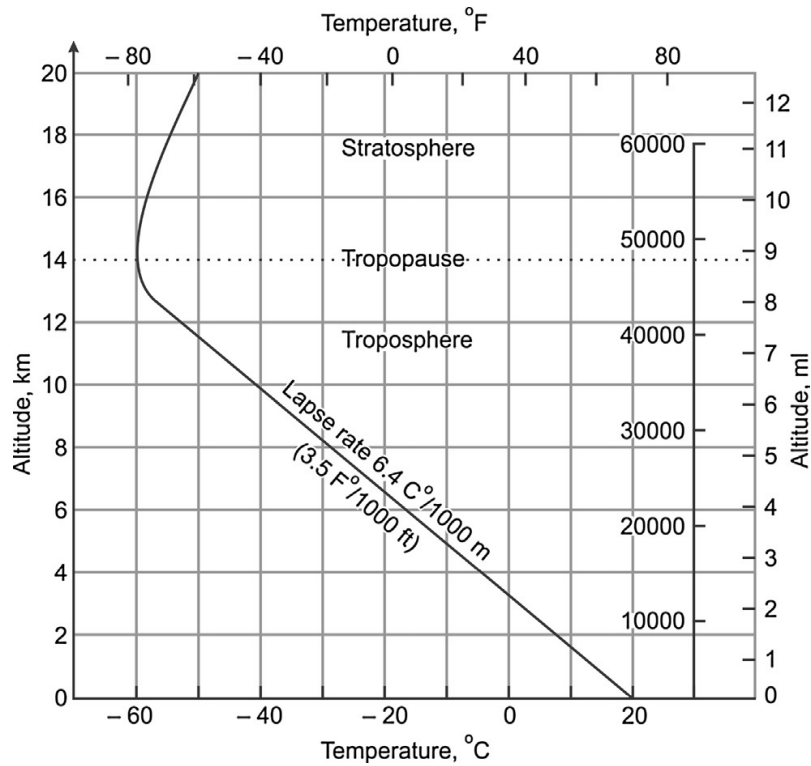


Figure- Lapse rates

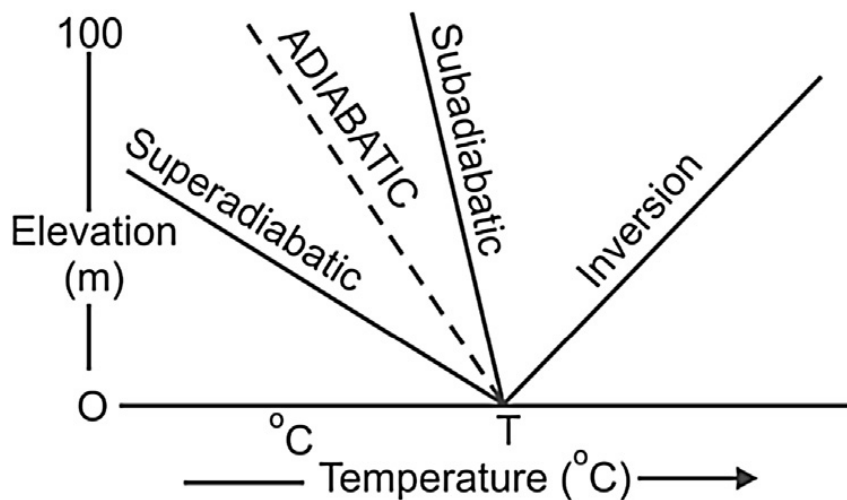


Figure - Summary of various adiabatic conditions

Humidity

Humidity is the concentration of water vapour present in air. Water vapour is the gaseous state of water and generally invisible to the human eye. Humidity indicates the possibility for precipitation, dew, or fog to be present. Humidity plays an important role for surface life.

The amount of water vapour needed to achieve saturation, increases as the temperature increases. As the temperature of a parcel of air decreases it will eventually reach the saturation point without adding or losing water mass. The amount of water vapour contained within a parcel of air can vary significantly.

Three primary measurements of humidity are widely used: absolute, relative and specific.

Absolute humidity describes the water content of air and is expressed in either grams per cubic metre or grams per kilogram.

Relative humidity, expressed as a percentage, indicates a present state of absolute humidity relative to a maximum humidity given the same temperature.

Specific humidity is the ratio of water vapour mass to total moist air parcel mass.

1. Absolute humidity

Absolute humidity is the total mass of water vapor present in a given volume or mass of air. It does not take temperature into consideration. Absolute humidity in the atmosphere ranges from near zero to roughly 30 grams per cubic metre when the air is saturated at 30 °C (86 °F). Absolute humidity is the mass of the water vapour (m_{H_2O}), divided by the volume of the air and water vapor mixture (V_{net}), which can be expressed as:

$$AH = m_{H_2O} / V_{net}$$

The absolute humidity changes as air temperature or pressure changes, if the volume is not fixed.

2. Relative humidity

The relative humidity (RH) of an air-water mixture is defined as the ratio of the partial pressure of water vapour (P_{H_2O}) in the mixture to the equilibrium vapor pressure of water ($P^*_{H_2O}$) over a flat surface of pure water at a given temperature:

$$RH = P_{H_2O} / P^*_{H_2O}$$

Relative humidity is normally expressed as a percentage; a higher percentage means that the air-water mixture is more humid. Relative humidity is an important metric used in weather forecasts and reports.

3. Specific humidity

Specific humidity (or moisture content) is the ratio of the mass of water vapor to the total mass of the air parcel. As temperature decreases, the amount of water vapor needed to reach saturation also decreases. As the temperature of a parcel of air becomes lower it will eventually reach the point of saturation without adding or losing water mass.

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