# CHAPTER-9

## Time – Series and Forecasting

A time series is a set of observations taken at a specified time usually at equal intervals.

**Utility of Time Series:**

1. Useful to know the past history of time series data.
2. Helps in planning the future operation.
3. To predict the future demand, weather conditions etc.
4. With reference to same reference period two or more time series can be compared.

**1- Methods of Measuring trends :**

**(1) Method of Semi-average :** (Method Based on linear relationship but fail in non-linear) under this method the data for which trend values are to be computed are divided into two equal parts and average are computed for both the parts. If there is odd number of years the value of middle year is omitted.



**Problem:** Calculated the trend values from the fllowing data by the method of semi-averages.

| **Year** | **1974** | **1975** | **1976** | **1977** | **1978** | **1979** | **1980** | **1981** |
|---|---|---|---|---|---|---|---|---|
| Sale | 10 | 11 | 13 | 8 | 14 | 12 | 9 | 14 |
| **Year** | **1982** | **1983** | **1984** | **1985** | **1986** | **1987** | **1988** | |
| Sale | 13 | 10 | 12 | 16 | 14 | 16 | 17 | |

**Solution:** We have odd number of years, so neglect the value of middle year 1981.

$$\text{Average for 1974 to 1980} = \frac{1}{7}(10+11+13+8+14+12+9) = \frac{77}{7} = 11$$

$$\text{Average for 1982 to 1988} = \frac{1}{7}(13+10+12+16+14+16+17) = \frac{98}{7} = 14$$

11 is ploted the mid year 1977 on the first half and 14 should be plotted against the mid year 1985 of the other half. These two points are jointed to get a trend line.

**Equation of trend line.**

The trend line passes through the point A(1977, 11) and the point B(1985, 14).

$$So\ y-11=\frac{14-11}{1985-1977}(x-1977)$$
$$\Rightarrow y-11=\frac{3}{8}(x-1977)$$
$$\Rightarrow y=11+0.375(x-1977)$$

**Method of Moving Averages :**

**Cast-I.** When period is odd.

Let the period of moving average be 3 years and items a, b, c, d, e. so, moving averages are :

Average of first three $=\frac{a+b+c}{3}$ is to be written against the mid year *b*.

Next year average $=\frac{b+c+d}{3}$ is to be written agains the mid yeear to c.

Next year average = $\frac{c+d+e}{3}$ is to be written against the mid year *d*.

**Case-II.** When period is even.

Let the period of moving average be 4-years then the average of Ist four figures will be placed between seconand third year. Similarly the average of $2^{nd}$ group of four years will be placed between third and fourth year.

These two moving averages will then be averaged and this average would be written as against the third year. This process is called centering of averages.

**Problem:** Calculate three yearly moving averages. Find a trend for the later.

| Year | 1985 | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 |
|---|---|---|---|---|---|---|---|---|
| Profit (in Rs.) | 15420 | 14470 | 15520 | 21020 | 26120 | 31950 | 35370 | 35670 |

**Solution:** Procedure

1. Put the original data of years and profits in colums of table.
2. Obtain 3 year moving totals starting from first year and put against the middle of year.
3. Now leaving Ist year and adding a successor year, find total of group and place it against the middle of years.
4. Keep on continuing unless all values are utilized.
5. Now divide each 3 years moving total by 3 to get the moving average.
6. Plot the moving average on a graph paper by taking years along x-axis and moving averages along y-axis.

| Years | Profit | 3 years moving total | 3 years moving average. |
|---|---|---|---|
| 1985 | 15420 | - | - |
| 1986 | 14470 | 45410 | 15136.7 |
| 1987 | 15520 | 51010 | 17003.3 |
| 1988 | 21020 | 51010 | 20886.7 |
| 1989 | 26120 | 62660 | 26363.3 |
| 1990 | 31950 | 79090 | 31146.7 |
| 1991 | 35370 | 93440 | 34330.0 |
| 1992 | 35670 | 102990 | |



**Problem:** Calculate trend by four years moving average of the data given below.

| **Year** | **1978** | **1979** | **1980** | **1981** | **1982** | **1983** | **1984** | **1985** |
|---|---|---|---|---|---|---|---|---|
| Production | 614 | 615 | 652 | 678 | 681 | 655 | 717 | 719 |
| | **1986** | **1987** | **1988** | | | | | |
| | 708 | 779 | 757 | | | | | |

**Solution :** Calculation of moving averages when period is even (say 4 years)

**Rule :**

**Step-1 :** Add the value of Ist four years and place total Ti, between the second and third year.

**Step-2 :** Leave the Ist year value and then add the values of next 4 years and place it in between third and fourth year continuing the process.

**Step-3 :** Divide 4 years moving total by 4 to get the 4 years-moving averages $A_1$ $A_2$ ………….

**Step-4 :** Add Ist two moving averages and divide it by 2 to get the moving average centered $C_1$ and place it against third year and so on.

| Year | Production | 4 years moving total | 4 year moving average | 4 year moving average centered |
|---|---|---|---|---|
| 1978 | 614 | | | |
| 1979 | 615 | 2559 | 639.50 | 648.125 |
| 1980 | 652 | 2626 | 656.50 | 661.500 |
| 1981 | 678 | 2731 | 682.75 | 674.625 |
| 1982 | 681 | 2772 | 693.00 | 687.875 |
| 1983 | 655 | 2799 | 699.75 | 696.375 |
| 1984 | 717 | 2923 | 730.75 | 715.250 |
| 1985 | 719 | 2923 | 740.75 | 735.750 |
| 1986 | 708 | 2963 | | |
| 1987 | 779 | | | |
| 1988 | 757 | | | |

*MEASUREMENT OF SECULAR TREND*

The principal methods of measuring trend fall into following categories:
1. Free Hand Curve methods
2. Method of Averages
3. Method of least squares

The *time series methods* are concerned with taking some observed historical pattern for some variable and projecting this pattern into the future using a mathematical formula. These methods do not attempt to suggest why the variable under study will take some future value. This limitation of the time series approach is taken care by the application of a causal method. The causal method tries to identify factors which influence the variable is some way or cause it to vary in some predictable manner. The two causal methods, regression analysis and correlation analysis, have already been discussed previously.

A few time series methods such as *freehand curves* and *moving averages* simply describe the given data values, while other methods such as *semi-average* and *least squares* help to identify a trend equation to describe the given data values.

**Freehand Method**

A freehand curve drawn smoothly through the data values is often an easy and, perhaps, adequate representation of the data. The forecast can be obtained simply by extending the trend line. A trend line fitted by the freehand method should conform to the following conditions:

(i) The trend line should be smooth- a straight line or mix of long gradual curves.

(ii) The sum of the vertical deviations of the observations above the trend line should equal the sum of the vertical deviations of the observations below the trend line.

(iii) The sum of squares of the vertical deviations of the observations from the trend line should be as small as possible.

(iv) The trend line should bisect the cycles so that area above the trend line should be equal to the area below the trend line, not only for the entire series but as much as possible for each full cycle.



**Example : Fit a trend line to the following data by using the freehand method.**

| Year | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 |
|---|---|---|---|---|---|---|---|---|
| Sales turnover : (Rs. in lakh) | 80 | 90 | 92 | 83 | 94 | 99 | 92 | 104 |

**Solution:**
presents the graph of turnover from 1991 to Forecast can simply by the trend

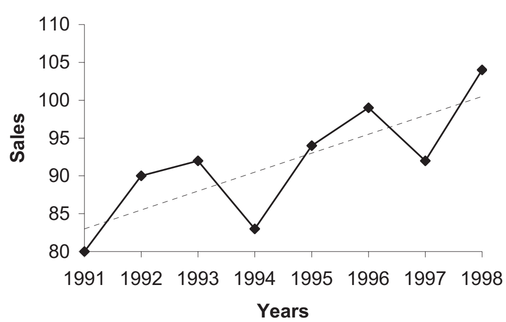



Figure freehand sales (Rs. in lakh) 1998. be obtained extending line.

**Fig. : Graph of Sales Turnover**

*Limitations of freehand method*

(i) This method is highly subjective because the trend line depends on personal judgement and therefore what happens to be a good-fit for one individual may not be so for another.

(ii) The trend line drawn cannot have much value if it is used as a basis for predictions.

(iii) It is very time-consuming to construct a freehand trend if a careful and conscientious job is to be done.

**Method of Averages**

The objective of smoothing methods into smoothen out the random variations due to irregular components of the time series and thereby provide us with an overall impression of the pattern of movement in the data over time. In this section, we shall discuss three smoothing methods.

(i) Moving averages

(ii) Weighted moving averages

(iii) Semi-averages



The data requirements for the techniques to be discussed in this section are minimal and these techniques are easy to use and understand.

*Moving Averages*

If we are observing the movement of some variable values over a period of time and trying to project this movement into the future, then it is essential to smooth out first the irregular pattern in the historical values of the variable, and later use this as the basis for a future projection. This can be done by calculating a series of moving averages.

This method is a subjective method and depends on the length of the period chosen for calculating moving averages. To remove the effect of cyclical variations, the period chosen should be an integer value that corresponds to or is a multiple of the estimated average length of a cycle in the series.

The moving averages which serve as an estimate of the next period's value of a variable given a period of length *n* is expressed as:

Moving average, $Ma_{t+1} = \frac{\sum\{D_t + D_{t-1} + D_{t-2}. + .. + D_{t-n+1}\}}{n}$

where $t$ = current time period
D = actual data which is exchanged each period
$n$ = length of time period

In this method, the term 'moving' is used because it is obtained by summing and averaging the values from a given number of periods, each time deleting the oldest value and adding a new value.

The limitation of this method is that it is highly subjective and dependent on the length of period chosen for constructing the averages. Moving averages have the following three limitations:

(i) As the size of $n$ (the number of periods averaged) increases, it smoothens the variations better, but it also makes the method less sensitive to real changes in the data.

(ii) Moving averages cannot pick-up trends very well. Since these are averages, it will always stay within past levels and will not predict a change to either a higher or lower level.

(iii) Moving average requires extensive records of past data.



**Example** Using three-yearly moving averages, determine the trend and short-term-error.

| Year | Production (in '000 tonnes) | Year | Production (in '000 tonnes) |
|---|---|---|---|
| 1987 | 21 | 1992 | 22 |
| 1988 | 22 | 1993 | 25 |
| 1989 | 23 | 1994 | 26 |
| 1990 | 25 | 1995 | 27 |
| 1991 | 24 | 1996 | 26 |

Solution: The moving average calculation for the first 3 years is:

$$\text{Moving average (year 1-3)} = \frac{21 + 22 + 23}{3} = 22$$

Similarly, the moving average calculation for the next 3 years is:

$$\text{Moving average (year 2-4)} = \frac{22 + 23 + 25}{3} = 22.33$$

A complete summary of 3-year moving average calculations is

**Table Calculation of Trend and Short-term Fluctuations**

| Year | ***Production Y*** | 3-Year Moving Total | 3-yearly Moving Average (Trend values $\hat{y}$ | Forecast Error (y- $\hat{y}$ ) |
|---|---|---|---|---|
| 1987 | 21 | - | - | - |
| 1988 | 22 | 66 | 22.00 | 0 |
| 1989 | 23 | 70 | 23.33 | -0.33 |
| 1990 | 25 | 72 | 24.00 | 1.00 |
| 1991 | 24 | 71 | 23.67 | 0.33 |
| 1992 | 22 | 71 | 23.67 | -1.67 |
| 1993 | 25 | 73 | 24.33 | 0.67 |

| | | | | |
|---|---|---|---|---|
| 1994 | 26 | 78 | 26.00 | 0 |
| 1995 | 27 | 79 | 26.33 | 0.67 |
| 1996 | 26 | - | - | - |

**Odd and Even Number of Years**

When the chosen period of length $n$ is an odd number, the moving average at year $i$ is centred on $i$, the middle year in the consecutive sequence of $n$ yearly values used to compute $i$. For instance with $n$ =5, $MA_3(5)$ is centred on the third year, $MA_4(5)$ is centred on the fourth year..., and $MA_9(5)$ is centred on the ninth year.

No moving average can be obtained for the first ($n$-1)/2 years or the last ($n$-1/2) year of the series. Thus for a 5-year moving average, we cannot make computations for the just two years or the last two years of the series.



When the chosen period of length $n$ is an even numbers, equal parts can easily be formed and an average of each part is obtained. For example, if $n$ = 4, then the first moving average $M_3$ (placed at period 3) is an average of the first four data values, and the second moving average $M_4$ (placed at period 4) is the average of data values 2 through 5). The average of $M_3$ and $M_4$ is placed at period 3 because it is an average of data values for period 1 through 5.

**Example** : Assume a four-yearly cycle and calculate the trend by the method of moving average from the following data relating to the production of tea in India.

| *Year* | *Production (million lbs)* | *Year* | *Production (million lbs)* |
|---|---|---|---|
| 1987 | 464 | 1992 | 540 |
| 1988 | 515 | 1993 | 557 |
| 1989 | 518 | 1994 | 571 |
| 1990 | 467 | 1995 | 586 |
| 1991 | 502 | 1996 | 612 |

Solution: The first 4-year moving average is:

$$MA_3(4) = \frac{464 + 515 + 518 + 467}{4} = \frac{1964}{4} = 491.00$$

This moving average is centred on the middle value, that is, the third year of the series. Similarly,

515 + 518 + 467+ 502 2002

MA (4) = $\dfrac{\quad}{4}$ = $\dfrac{\quad}{4}$ = 500.50

This moving average is centred on the fourth year of the series.

the data along with the computations of 4-year moving averages.

**Table : Calculation of Trend and Short-term Fluctuations**

| Year | Production (mm lbs) | 4-yearly Moving Totals | 4-Yearly Moving Average | 4-Yearly Moving Average Centred |
|---|---|---|---|---|
| 1987 | 464 | - | | - - |
| 1988 | 515 | - - | | - |
| | | 1964 | | |
| 1989 | 518 | 491.00 | | 495.75 |
| | | 2002 | | |
| 1990 | 467 | 500.50 | | 503.62 |
| | | 2027 | | |
| 1991 | 502 | 506.75 | | 511.62 |
| | | 2066 | | |
| 1992 | 540 | 516.50 | | 529.50 |
| | | 2170 | | |
| 1993 | 557 | 542.50 | | 553.00 |
| | | 2254 | 563.50 | |
| 1994 | 571 | | | 572.00 |
| | | 2326 | | - |
| 1995 | 586 | 581.50 | | - - |
| 1996 | 612 | - - | | - |



**Weighted Moving Averages**

In moving averages, each observation is given equal importance (weight). However, different values may be assigned to calculate a weighted average of the most recent *n* values. Choice of weights is somewhat arbitrary because there is no set formula to determine them. In most cases, the most recent observation receives the most weightage, and the weight decreases for older data values.

A weighted moving average may be expressed mathematically as

$$\text{Weighted moving average} = \frac{\Sigma(\text{Weight for period } n)\ (\text{Data value in period } n)}{\Sigma\text{Weights}}$$

**Example :** Vaccum cleaner sales for 12 months is given below. The owner of the supermarket decides to forecast sales by weighting the past three months as follows:

| Weight Applied | Month |
|---|---|
| 3 | Last month |
| 2 | Two months ago |
| 1 | Three months ago |
| 6 | |

| Month | : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Actual sales (in units) | : | 10 | 12 | 13 | 16 | 19 | 23 | 26 | 30 | 28 | 18 | 16 | 14 |

**Solution:** The results of 3-month weighted average are shown in Table .

$$\text{Forecast for the Current month} = \frac{3 \times \text{Sales last month} + 2 \times \text{Sales two months ago} + 1 \times \text{Sales three months ago}}{6}$$

**Table : Weighted Moving Average**

| Month | Actual Sales | Three-month Weighted Moving Average |
|---|---|---|
| 1 | 10 | - |
| 2 | 12 | - |
| 3 | 13 | - |
| 4 | 16 | $\frac{1}{6}[3\times 13)+(2\times 12)+1\times 10]=\frac{121}{6}$ |
| 5 | 19 | $\frac{1}{6}[3\times 16)+(2\times 13)+1\times 12]=\frac{141}{3}$ |
| 6 | 23 | $\frac{1}{6}[3\times 19)+(2\times 16)+1\times 13]=17$ |
| 7 | 26 | $\frac{1}{6}[3\times 23)+(2\times 19)+1\times 16]=\frac{201}{2}$ |
| 8 | 30 | $\frac{1}{6}[3\times 26)+(2\times 23)+1\times 19]=\frac{235}{6}$ |
| 9 | 28 | $\frac{1}{6}[3\times 30)+(2\times 26)+1\times 23]=\frac{271}{2}$ |
| 10 | 18 | $\frac{1}{6}[3\times 28)+(2\times 30)+1\times 26]=\frac{289}{3}$ |
| 11 | 16 | $\frac{1}{6}[3\times 18)+(2\times 28)+1\times 30]=\frac{231}{3}$ |
| 12 | 14 | $\frac{1}{6}[3\times 16)+(2\times 18)+1\times 28]=\frac{182}{3}$ |



**Example :** A food processor uses a moving average to forecast next month's demand. Past actual demand( in units) is shown below:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Month | : | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 |
| Actual demand (in units) | : | 105 | 106 | 110 | 110 | 114 | 121 | 130 | 128 | 137 |

(a) Compute a simple five-month moving average to forecast demand for month 52.

(b) Compute a weighted three-month moving average where the weights are highest for the latest months and descend in order of 3, 2, 1.

**Solution:** Calculation for five-month moving average are shown in Table 7.4.

| Month | Actual Demand | 5-month Moving Total | 5-month Moving Average |
|---|---|---|---|
| 43 | 105 | - | - |
| 44 | 106 | - | - |
| 45 | 110 | 545 | 109.50 |
| 46 | 110 | 561 | 112.2 |
| 47 | 114 | 585 | 117.0 |
| 48 | 121 | 603 | 120.6 |

| | | | |
|---|---|---|---|
| 49 | 130 | 630 | 126.0 |
| 50 | 128 | - | - |
| 51 | 137 | - | - |

(a) Five-month average demand for month 52 is

$$\frac{\Sigma x}{\text{Number of periods}} = \frac{114 + 121 + 130 + 128 + 137}{5} = 126 \text{ units}$$

(b) Weighted three-month average as per weights is as follows:

$$MA_{Wt} = \frac{\Sigma \text{ Weight} \times \text{Data value}}{\Sigma \text{ weight}}$$

Where Month Weight × Value = Total

| Month | Weight × Value | | Total |
|---|---|---|---|
| 51 | 3 × 137 | = | 141 |
| 50 | 2 × 128 | = | 256 |
| 49 | 1 × 130 | = | 130 |
| | 6 | | 797 |

$MA_{WT} = \frac{797}{6} = 133$ units



**Semi-Average Method**

The semi-average method permits us to estimate the slope and intercept of the trend the quite easily if a linear function will adequately described the data. The procedure is simply to divide the data into two parts and compute their respective arithmetic means. These two points are plotted corresponding to their midpoint of the class interval covered by the respective part and then these points are joined by a straight line, which is the required trend line. The arithmetic mean of the first part is the intercept value, and the slope is determined by the ratio of the difference in the arithmetic mean of the number of years between them, that is, the change per unit time. The resultant is a time series of the form : $\hat{y} = a + bx$ . The $\hat{y}$ is the calculated trend value and $a$ and $b$ are the intercept and slope values respectively. The equation should always be stated completely with reference to the year where x =0 and a description of the units of $x$ and $y$.

The semi-average method of developing a trend equation is relatively easy to commute and may be satisfactory if the trend is linear. If the data deviate much from linearity, the forecast will be biased and less reliable.

**Example :** Fit a trend line to the following data by the method of semi-average and forecast the sales for the year 2002.

| *Year* | *Sales of Firm (thousand units)* | *Year* | *Sales of Firm (thousand units)* |
|---|---|---|---|
| 1993 | 102 | 1997 | 108 |
| 1994 | 105 | 1998 | 116 |
| 1995 | 114 | 1999 | 112 |
| 1996 | 110 | | |

**Solution:** Since number of years are odd in number, therefore divide the data into equal parts (A and B) of 3 years ignoring the middle year (1996). The average of part A and B is

$$\bar{y}_A = \frac{102 + 105 + 114}{3} = \frac{321}{3} = 107 \text{ units}$$

$$\bar{y}_B = \frac{108 + 116 + 112}{3} = \frac{336}{2} = 112 \text{ units}$$

Part A is centred upon 1994 and part B on 1998. Plot points 107 and 112 against their middle years, 1994 and 1998. By joining these points, we obtain the required trend line as shown Fig.

The line can be extended and be used for prediction.

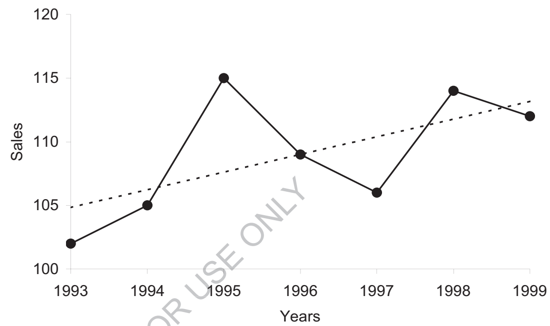





*Fig : Trend Line by the Method of Semi-Average*

To calculate the time-series $\hat{y} = a + bx$, we need

$$\text{Slope } b = \frac{\Delta y}{\Delta x} = \frac{\text{Change in sales}}{\text{Change in year}}$$

$$= \frac{112 - 107}{1998 - 1994} = \frac{5}{4} = 1.25$$

Intercept = a = 107 units at 1994

Thus, the trend line is : $\hat{y} = 107 + 1.25x$

Since 2002 is 8 year distant from the origin (1994), therefore we have

$$\hat{y} = 107 + 1.25(8) = 117$$

**Exponential Smoothing Methods**

Exponential smoothing is a type of moving-average forecasting technique which weighs past data in an exponential manner so that the most recent data carries more weight in the moving average. Simple exponential smoothing makes no explicit adjustment for trend effects whereas adjusted exponential smoothing does take trend effect into account (see next section for details).

**Simple Exponential Smoothing**

forecast.

$$F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1}) = (1-\alpha)F_{t-1} + \alpha D_{t-1} \qquad \ldots(7.1)$$

Where $F_t$ = current period forecast

$F_{t-1}$ = last period forecast

$\alpha$ = a weight called smoothing constant ($0 \le \alpha \le 1$)

$D_{t-1}$ = last period actual demand

From Eqn. (7.1), we may notice that each forecast is simply the previous forecast plus some correction for demand in the last period. If demand was above the last period forecast the correction will be positive, and if below it will negative.

When *smoothing constant* $\alpha$ is low, more weight is given to past data, and when it is high, more weight is given to recent data. When $\alpha$ is equal to 0.9, then 99.99 per cent of the forecast value is determined by the four most recent demands. When $\alpha$ is as low as 0.1, only 34.39 per cent of the average is due to these last 4 periods and the smoothing effect is equivalent to a 19-period arithmetic moving average.

If $\alpha$ were assigned a value as high as 1, each forecast would reflect total adjustment to the recent demand and the forecast would simply be last period's actual demand, that is, $F_t = 1.0D_{t-1}$. Since demand fluctuations are typically random, the value of $\alpha$ is generally kept in the range of 0.005 to 0.30 in order to 'smooth' the forecast. The exact value depends upon the response to demand that is best for the individual firm.

The following table helps illustrate this concept. For example, when $\alpha = 0.5$, we can see that the new forecast is based on demand in the last three or four periods. When $\alpha = 0.1$, the forecast places little weight on recent demand and takes a 19-period arithmetic moving average.

| *Smoothing Constant* | *Weight Assigned to* | | | | |
|---|---|---|---|---|---|
| | *Most Recent Period* ($\alpha$) | *2nd Most Recent Period* $\alpha(1-\alpha)$ | *3rd Most Recent Period* $\alpha(1-\alpha)^2$ | *4th Most Recent Period* $\alpha(1-\alpha)^3$ | *5th Most Recent Period* $\alpha(1-\alpha)^4$ |
| $\alpha = 0.1$ | 0.1 | 0.09 | 0.081 | 0.073 | 0.066 |
| $\alpha = 0.5$ | 0.5 | 0.25 | 0.125 | 0.063 | 0.031 |



### *Selecting the smoothing constant*

The exponential smoothing approach is easy to use and it has been successfully applied by banks, manufacturing companies, wholesalers, and other organizations. The appropriate value of the smoothing constant, $\alpha$, however, can make the difference between an accurate and an inaccurate forecast. In picking a value for the smoothing constant, the objective is to obtain the most accurate forecast.

The correct $\alpha$-value facilitates scheduling by providing a reasonable reaction to demand without incorporating too much random variation. An approximate value of $\alpha$ which is equivalent to an arithmetic moving average, in terms of degree of smoothing, can be estimated as: $\alpha = 2\ (n+1)$. The accuracy of a forecasting model can be determined by comparing the forecasting values with the actual or observed values.

The forecast error is defined as:

Forecast error = Actual values – Forecasted values

One measure of the overall forecast error for a model is the *mean absolute deviation (MAD)*. This is computed by taking the sum of the absolute values of the individual forecast errors and dividing by the number of periods $n$ of data.

$$MAD = \frac{\Sigma|\text{Forecast errors}|}{n}$$

where Standard deviation σ = 1.25 MAD

The exponential smoothing method also facilities continuous updating of the estimate of MAD. The current $MAD_t$ is given by

$MAD_t = \alpha|\text{Actual values- Forecasted values}| + (1-\alpha)\, MAD_{t-1}$

Higher values of smoothing constant α make the current MAD more responsive to current forecast errors.

**Example :** A firm uses simple exponential smoothing with α =0.1 to forecast demand. The forecast for the week of February 1 was 500 units whereas actual demand turned out to be 450 units.

(a) Forecast the demand for the week of February 8.

(b) Assume the actual demand during the week of February 8 turned out to be 505 units. Forecast the demand for the week of February 15. Continue forecasting through March 15, assuming that subsequent demands were actually 516, 488, 467, 554 and 510 units.

Solution: Given $F_{t-1} = 500$, $D_{t-1} = 450$, and α = 0.1

(a) $F_t = F_{t-1} - \alpha(D_{t-1} - F_{t-1}) = 500 + 0.1(450\text{-}500) = 495$ units

(b) Forecast of demand for the week of February 15 is shown in Table 7.5

**Table : Forecast of Demand**

| Week | Demand $D_{t-1}$ | Old Forecast $F_{t-1}$ | Forecast Error $(D_{t-1} - F_{t-1})$ | Correction $\alpha(D_{t-1} - F_{t-1})$ | New Forecast ($F_t$) $F_{t-1} + \alpha(D_{t-1} - F_{t-1})$ |
|---|---|---|---|---|---|
| Feb. 1 | 450 | 500 | -50 | -5 | 495 |
| Feb. 8 | 505 | 495 | 10 | 1 | 496 |
| Feb. 15 | 516 | 496 | 20 | 2 | 498 |
| Feb. 22 | 488 | 498 | -10 | -1 | 497 |
| Mar. 1 | 467 | 497 | -30 | -3 | 494 |
| Mar. 8 | 554 | 494 | 60 | 6 | 500 |
| Mar. 15 | 510 | 500 | 10 | 1 | 501 |



If no previous forecast value is known, the old forecast starting point may be estimated or taken to be an average of some preceding periods.

**Example :** A hospital has used a 9 month moving average forecasting method to predict drug and surgical inventory requirements. The actual demand for one item is shown in the table below. Using the previous moving average data, convert to an exponential smoothing forecast for month 33.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Month | : | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| Demand (in units) | : | 78 | 65 | 90 | 71 | 80 | 101 | 84 | 60 | 73 |

Solution: The moving average of a 9-month period is given by

$$MA = \frac{\sum \text{Demand (x)}}{\text{Number of periods}} = \frac{78 + 65 \ldots + 73}{9} = 78$$

Assume $F_{t-1} = 78$. Therefore, estimated $\alpha = \frac{2}{n+1} = \frac{2}{9+1} = 0.2$

Thus, $F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1}) = 78 + 0.2\,(73 - 78) = 77$ units

*Methods of least square*

The trend project method fits a trend line to a series of historical data points and then projects the line into the future for medium-to-long range forecasts. Several mathematical trend equations can be developed (such as exponential and quadratic), depending upon movement of time-series data.

**Reasons to study trend:** A few reasons to study trends are as follows:

1. The study of trend allows us to describe a historical pattern so that we may evaluate the success of previous policy.
2. The study allows us to use trends as an aid in making intermediate and long-range forecasting projections in the future.
3. The study of trends helps us to isolate and then eliminate its influencing effects on the time-series model as a guide to short-run (one year or less) forecasting of general business cycle conditions.

*Linear Trend Model*

If we decide to develop a linear trend line by a precise statistical method, we can apply the *least squares method.* A least squares line is described in terms of its *y*-intercept (the height at which it intercepts the *y*-axis) and its slope (the angle of the line). If we can compute the *y*-intercept and slope, we can express the line with the following equation

$$\hat{y} = a + bx$$

where $\hat{y}$ = predicted value of the dependent variable

a = y-axis intercept

b = slope of the regression line (or the rate of change in y for a given change in x)

x = independent variable (which is *time* in this case)

Least squares is one of the most widely used methods of fitting trends to data because it yields what is mathematically described as a 'line of best fit'. This trend line has the properties that (i) the summation of all vertical deviations about it is zero, that is, $\Sigma(y-\hat{y}) = 0$, (ii) the summation f all vertical deviations squared is a minimum, that is, $\Sigma(y-\hat{y})$ is least, and (iii) the line goes through the mean values of variables x and y. For linear equations, it is found by the simultaneous solution for $a$ and $b$ of the two normal equations:

$$\Sigma y = na + b\Sigma x \text{ and } \Sigma xy = a\Sigma x + b\Sigma x^2$$



Where the data can be coded so that $\sum x = 0$, two terms in three equations drop out and we have $\Sigma y = na$ and $\Sigma xy = b\Sigma x^2$

Coding is easily done with time-series data. For coding the data, we choose the centre of the time period as $x = 0$ and have an equal number of plus and minus periods on each side of the trend line which sum to zero.

Alternately, we can also find the values of constants $a$ and $b$ for any regression line as:

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(\bar{x})^2} \text{ and } a = \bar{y} - b\bar{x}$$

**Example** : Below are given the figures of production (in thousand quintals) of a sugar factory:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Year | : | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 |
| Production | : | 80 | 90 | 92 | 83 | 94 | 99 | 92 |

(a) Fit a straight line trend to these figures.

(b) Plot these figures on a graph and show the trend line.

(c) Estimate the production in 2001.

Solution: (a) Using normal equations and the sugar production data we can compute constants $a$ and $b$ as shown in Table 7.6:

**Table : Calculations for Least Squares Equation**

| *Year* | *Time Period (x)* | *Production (x)* | $x^2$ | *xy* | *Trend Values* $\bar{y}$ |
|---|---|---|---|---|---|
| 1992 | 1 | 80 | 1 | 80 | 84 |
| 1993 | 2 | 90 | 4 | 180 | 86 |
| 1994 | 3 | 92 | 9 | 276 | 88 |
| 1995 | 4 | 83 | 16 | 332 | 90 |
| 1996 | 5 | 94 | 25 | 470 | 92 |
| 1997 | 6 | 99 | 36 | 594 | 94 |
| 1998 | 7 | 92 | 49 | 644 | 96 |
| **Total** | **28** | **630** | **140** | **2576** | |



$$\bar{x} = \frac{\sum x}{n} = \frac{28}{7} = 4 \, , \; \bar{y} = \frac{\sum y}{n} = \frac{630}{7} = 90$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(\bar{x})^2} = \frac{2576 - 7(4)(90)}{140 - 7(4)^2} = \frac{56}{28} = 2$$

$$a = \bar{y} - b\bar{x} = 90 - 2(4) = 82$$

Therefore, linear trend component for the production of sugar is:

$$\hat{y} = a + bx = 82 + 2x$$

The slope $b = 2$ indicates that over the past 7 years, the production of sugar had an average growth of about 2 thousand quintals per year.

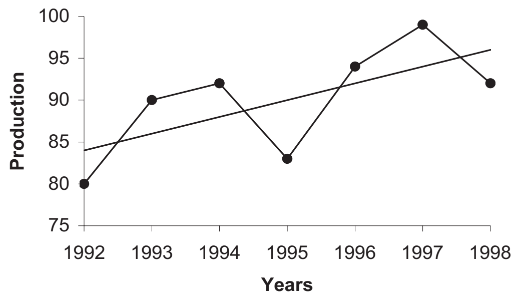



**Fig. : Linear Trend for Production of Sugar**

(b) Plotting points on the graph paper, we get an actual graph representing production of sugar over the past 7 years. Join the point $a$ = 82 and $b$ = 2 (corresponds to 1993) on the graph we get a trend line as shown in Fig. 7.4.

(c) The production of sugar for year 2001 will be

$$\hat{y} = 82 + 2\,(10) = 102 \text{ thousand quintals}$$



**Parabolic Trend Model**

The curvilinear relationship for estimating the value of a dependent variable $y$ from an independent variable $x$ might take the form

$$\hat{y} = a + bx + cx^2$$

This trend line is called the *parabola.*

For a non-linear equation $\hat{y} = a + bx - cx^2$, the values of constants $a, b$, and $c$ can be determined by solving three normal equations.

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

When the data can be coded so that $\Sigma x = 0$ and $\Sigma x^3 = 0$, two term in the above expressions drop out and we have

$$\Sigma y = na + c\Sigma x^2$$

$$\Sigma xy = b\Sigma x^2$$

$$\Sigma x^2 y = a\Sigma x^2 + c\Sigma x^4$$

To find the exact estimated value of the variable *y*, the values of constants *a, b,* and *c* need to be calculated. The values of these constants can be calculated by using the following shortest method:

$$a = \frac{\sum y - c\sum x^2}{n};\ b = \frac{\sum xy}{\sum x^2} \text{ and } c = \frac{n\sum x^2 y - \sum x^2 \sum y}{n\sum x^4 - (\sum x^2)^2}$$

**Example** : The prices of a commodity during 1999-2004 are given below. Fit a parabola to these data. Estimate the price of the commodity for the year 2005.

| Year | Price | Year | Price |
|---|---|---|---|
| 1999 | 100 | 2002 | 140 |
| 2000 | 107 | 2003 | 181 |
| 2001 | 128 | 2004 | 192 |



Also plot the actual and trend values on a graph.

**Solution:** To fit a parabola $\hat{y} = a + bx + cx^2$, the calculations to determine the values of constants *a*, *b,* and *c* are shown in Table .

**Table : Calculations for Parabola Trend Line**

| *Year* | *Time Scale (x)* | *Price (y)* | $x^2$ | $x^3$ | $x^4$ | *xy* | $x^2y$ | *Trend Values* ($\hat{y}$) |
|---|---|---|---|---|---|---|---|---|
| 1999 | -2 | 100 | 4 | -8 | 16 | -200 | 400 | 97.72 |
| 2000 | -1 | 107 | 1 | -1 | 1 | -107 | 107 | 110.34 |
| 2001 | 0 | 128 | 0 | 0 | 0 | 0 | 0 | 126.68 |
| 2002 | 1 | 140 | 1 | 1 | 1 | 140 | 140 | 146.50 |
| 2003 | 2 | 181 | 4 | 8 | 16 | 362 | 724 | 169.88 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2004 | 3 | 192 | 9 | 27 | 81 | 576 | 1728 | 196.82 |
| | **3** | **848** | **19** | **27** | **115** | **771** | **3099** | **847.94** |

(i) $\Sigma y = na - b\Sigma x + c\Sigma x^2$

$848 = 6a + 3b + 19c$

(ii) $\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$771 = 3a + 19b + 27c$

(iii) $\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^2 + c\Sigma x^4$

$3099 = 19a + 27b + 115c$

Eliminating a from eqns. (i) and (ii), we get

(iv) $694 = 35b + 35c$

Eliminating a from eqns. (ii) and (iii), we get

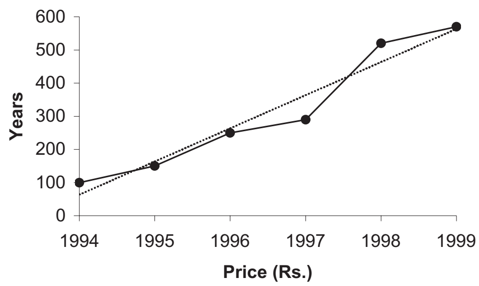
(v) $5352 = 280b + 168c$

Solving eqns. (iv) and (v) for $b$ and $c$ we get $b$ =18.04 and $c$ = 1.78. Substituting values of $b$ and $c$ in eqn. (i), we get $a$ = 126.68.

Hence, the required non-linear trend line becomes

$$y = 126.68 + 18.04x + 1.78x^2$$

Several trend values as shown in Table can be obtained by putting $x$ = -2, -1, 0, 1, 2 and 3 in the trend line. The trend values are plotted on a graph paper. The graph is shown in Fig. .




**Exponential Trend Model**

When the given values of dependent variable $y$ from approximately a geometric progression while the corresponding independent variable $x$ values form an arithmetic progression, the relationship between variables $x$ and $y$ is given by an exponential function, and the best fitting curve is said to describe the *exponential trend.* Data from the fields of biology, banking, and economics frequently exhibit such a trend. For example, growth of bacteria, money accumulating at compound interest, sales or earnings over a short period, and so on, follow exponential growth.



The characteristics property of this law is that the rate of growth, that is, the rate of change of $y$ with respect to $x$ is proportional to the values of the function. The following function has this property.

$$y = ab^{cx}, \ a > 0$$

The letter $b$ is a fixed constant, usually either 10 or $e$, where $a$ is a constant to be determined from the data.

To assume that the law of growth will continue is usually unwarranted, so only short range predictions can be made with any considerable degree or reliability.

If we take logarithms (with base 10) of both sides of the above equation, we obtain

$$\text{Log } y = \log a + (c \log b)\, x$$

For $b$ =10, log $b$ =1, but for $b$=e, log $b$ =0.4343 (approx.). In either case, this equation is of the form $y' = c + dx$

Where $y' = \log y$, $c = \log a$, and $d = c \log b$.

Equation represents a straight line. A method of fitting an exponential trend line to a set of observed values of $y$ is to fit a straight trend line to the logarithms of the $y$-values.

In order to find out the values of constants $a$ and $b$ in the exponential function, the two normal equations to be solved are

$\Sigma \log y = n \log a + \log b \Sigma x$

$\Sigma x \log y = \log a \Sigma x + \log b \Sigma x^2$

When the data is coded so that $\Sigma x = 0$, the two normal equations become

$\Sigma \log y = n \log a$ or $\log a = \frac{1}{n} \Sigma \log y$

and $\Sigma x \log y = \log b\, \Sigma x^2$ or $\log b = \frac{\sum x \log y}{\sum x^2}$



Coding is easily done with time-series data by simply designating the center of the time period as $x$ =0, and have equal number of plus and minus period on each side which sum to zero.

**Example** : The sales (Rs. In million) of a company for the years 1995 to 1999 are:

| | | | | | |
|---|---|---|---|---|---|
| Year : | 1995 | 1996 | 1997 | 1998 | 1999 |
| Sales : | 1.6 | 4.5 | 13.8 | 40.2 | 125.0 |

Find the exponential trend for the given data and estimate the sales for 2002.

**Solution:** The computational time can be reduced by coding the data. For this consider $u = x$-3. The necessary computations are shown in Table 7.8.

**Table : Fitting the Exponential Trend Line**

| Year | Time Period $x$ | $u=x$-3 | $u^2$ | Sales $y$ | Log $y$ | $u$ log $y$ |
|---|---|---|---|---|---|---|
| 1995 | 1 | -2 | 4 | 1.60 | 0.2041 | -0.4082 |
| 1996 | 2 | -1 | 1 | 4.50 | 0.6532 | -0.6532 |
| 1997 | 3 | 0 | 0 | 13.80 | 1.1390 | 0 |
| 1998 | 4 | 1 | 1 | 40.20 | 1.6042 | 1.6042 |
| 1999 | 5 | 2 | 4 | 125.00 | 2.0969 | 4.1938 |
| | | | **10** | | **5.6983** | **4.7366** |

$$\log a = \frac{1}{n}\ \Sigma \log y = \frac{1}{5}(5.6983) = 1.1397$$

$$\log b = \frac{\sum u \log y}{\sum u^2} = \frac{4.7366}{10} = 0.4737$$

Therefore $\log y = \log a + (x+3) \log b = 1.1397 + 0.4737x$

For sales during 2002, $x$ =3, and we obtain

$\log y = 1.1397 + 0.4737\ (3) = 2.5608$

$y$ = antilog (2.5608) = 363.80



**Changing the Origin and Scale of Equations**

When a moving average or trend value is calculated it is assumed to be centred in the middle of the month (fifteenth day) or the year (July 1). Similarly, the forecast value is assumed to be centred in the middle of the future period. However, the reference point (origin) can be shifted, or the units of variables $x$ and $y$ are changed to monthly or quarterly values it desired. The procedure is as follows:

(i) Shift the origin, simply by adding or subtracting the desired number of periods from independent variable $x$ in the original forecasting equation.

(ii) Change the time units from annual values to monthly values by dividing independent variable $x$ by 12.

(iii) Change the $y$ units from annual to monthly values, the entire right-hand side of the equation must be divided by 12.

**Example** : The following forecasting equation has been derived by a least-squares method:

$\hat{y}$ =10.27 + 1.65$x$ (Base year:1992; $x$ = years; $y$ = tonnes/year)

Rewrite the equation by

(a) shifting the origin to 1997.

(b) expressing $x$ units in months, retaining $y$ n tonnes/year.

(c) expressing $x$ units in months and $y$ in tonnes/month.

**Solution:** (a) Shifting of origin can be done by adding the desired number of period 5(=1997-1992) to $x$ in the given equation. That is

$$\hat{y} = 10.27 + 1.65\,(x + 5) = 18.52 + 1.65x$$

where 1997 = 0, $x$ = years, $y$ = tonnes/year

(b) Expressing $x$ units in months

$$\hat{y} = 10.27 + \frac{1.65x}{12} = 10.27 + 0.14x$$

where July 1, 1992 = 0, $x$ = months, $y$ = tonnes/year

(c) Expressing $y$ in tonnes/month, retaining $x$ months.

$$\hat{y} = \frac{1}{12} = (10.27 + 0.14x) = 0.86+0.01x$$

where July 1, 1992 = 0, $x$ = months, $y$ = tones/month

**Remarks**

1. If both $x$ and $y$ are to be expressed in months together, then divide constant 'a' by 12 and constant 'b' by 24. It is because data are sums of 12 months. Thus monthly trend equation becomes.

Linear trend : $\hat{y} = \frac{a}{12} + \frac{b}{24}x$

Parabolic trend : $\hat{y} = \frac{a}{12} + \frac{b}{144}x + \frac{c}{1728}x^2$

But if data are given as monthly averages per year, then value of '*a*' remains unchanged '*b*' is divided by 12 and '*c*' by 144.

2. The annual trend equation can be reduced to quarterly trend equation as :

$$\hat{y} = \frac{a}{4} + \frac{b}{4 \times 12}x = \frac{a}{4} + \frac{b}{48}x$$

**SEASONAL VARIATIONS**

If the time series data are in terms of annual figures, the seasonal variations are absent. These variations are likely to be present in data recorded on quarterly or monthly or weekly or daily or hourly basis. As discussed earlier, the seasonal variations are of periodic in nature with period less than or equal to one year. These variations reflect the annual repetitive pattern of the economic or business activity of any society. The main objectives of measuring seasonal variations are:



(i) To understand their pattern.
(ii) To use them for short-term forecasting or planning.
(iii) To compare the pattern of seasonal variations of two or more time series in a given period or of the same series in different periods.
(iv) To eliminate the seasonal variations from the data. This process is known as *deseasonalisation* of data.

**Methods of Measuring Seasonal Variations**

The measurement of seasonal variation is done by isolating them from other components of a time series. There are four methods commonly used for the measurement of seasonal variations. These method are:

1. Method of Simple Averages
2. Ratio to Trend Method
3. Ratio to Moving Average Method

4. Method of Line Relatives

Note: In the discussion of the above methods, we shall often assume a multiplicative model. However, with suitable modifications, these methods are also applicable to the problems based on additive model.

**Method of Simple Averages**

This method is used when the time series variable consists of only the seasonal and random components. The effect of taking average of data corresponding to the same period (say 1st quarter of each year) is to eliminate the effect of random component and thus, the resulting averages consist of only seasonal component. These averages are then converted into seasonal indices, as explained in the following examples.

**Example**



Assuming that trend and cyclical variations are absent compute the seasonal index for each month of the following data of sales (in Rs. '000) of a company.

| *Year* | *Jan* | *Feb* | *Mar* | *Apr* | *May* | *Jun* | *Jul* | *Aug* | *Sep* | *Oct* | *Nov* | *Dec* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1987 | 46 | 45 | 44 | 46 | 45 | 47 | 46 | 43 | 40 | 40 | 41 | 45 |
| 1988 | 45 | 44 | 43 | 46 | 46 | 45 | 47 | 42 | 43 | 42 | 43 | 44 |
| 1989 | 42 | 41 | 40 | 44 | 45 | 45 | 46 | 43 | 41 | 40 | 42 | 45 |

Solution

**Calculation Table**

| *Year* | *Jan* | *Feb* | *Mar* | *Apr* | *May* | *Jun* | *Jul* | *Aug* | *Sep* | *Oct* | *Nov* | *Dec* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1987 | 46 | 45 | 44 | 46 | 45 | 47 | 46 | 43 | 40 | 40 | 41 | 45 |
| 1988 | 45 | 44 | 43 | 46 | 46 | 45 | 47 | 42 | 43 | 42 | 43 | 44 |
| 1989 | 42 | 41 | 40 | 44 | 45 | 45 | 46 | 43 | 41 | 40 | 42 | 45 |
| Total | 133 | 130 | 127 | 136 | 136 | 137 | 139 | 128 | 124 | 122 | 126 | 134 |
| A*t* | 44.3 | 43.3 | 42.3 | 45.3 | 45.3 | 45.7 | 46.3 | 42.7 | 41.3 | 40.7 | 42.0 | 44.7 |
| S.*I.* | 101.4 | 99.1 | 96.8 | 103.7 | 103.7 | 104.6 | 105.9 | 97.7 | 94.5 | 93.1 | 96.1 | 102.3 |

In the above table, A denotes the average and *S.I* the seasonal index for a particular month of various years. To calculate the seasonal index, we compute grand average $G$, given by $G = \frac{\sum A_i}{12} = \frac{523}{12} = 43.7$. Then the seasonal index for a particular month is given by $S.I. = \frac{A_i}{G} \times 100$.

Further, $\Sigma S.I. = 11998.9 \neq 1200$. Thus, we have to adjust these values such that their total is 1200. This can be done by multiplying each figure by $\frac{1200}{1198.9}$. The resulting figures are the adjusted seasonal indices, as given below:

| *Jan* | *Feb* | *Mar* | *Apr* | *May* | *Jun* | *Jul* | *Aug* | *Sep* | *Oct* | *Nov* | *Dec* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 101.5 | 99.2 | 96.9 | 103.8 | 103.8 | 104.7 | 106.0 | 97.8 | 94.6 | 93.2 | 96.2 | 102.3 |

Remarks: The total equal to 1200, in case of monthly indices and 400, in case of quarterly indices, indicate that the ups and downs in the time series, due to seasons, neutralise themselves within that year. It is because of this that the annual data are free from seasonal component.



**Example**

Compute the seasonal index from the following data by the method of simple averages.

| *Year* | *Quarter* | *Y* | *Year* | *Quarter* | *Y* | *Year* | *Quarter* | *Y* |
|---|---|---|---|---|---|---|---|---|
| 1980 | I | 106 | 1982 | I | 90 | 1984 | I | 80 |
| | II | 124 | | II | 112 | | II | 104 |
| | III | 104 | | III | 101 | | III | 95 |
| | IV | 90 | | IV | 85 | | IV | 83 |
| 1981 | I | 84 | 1983 | I | 76 | 1985 | I | 104 |
| | II | 114 | | II | 94 | | II | 112 |
| | III | 107 | | III | 91 | | III | 102 |
| | IV | 88 | | IV | 76 | | IV | 84 |

**Solution**

**Calculation of Seasonal Indices**

| Years | Ist Qr | 2nd Qr | 3rd Qr | 4th Qr |
|---|---|---|---|---|
| 1980 | 106 | 124 | 104 | 90 |
| 1981 | 84 | 114 | 107 | 88 |
| 1982 | 90 | 112 | 101 | 85 |
| 1983 | 76 | 94 | 91 | 76 |
| 1984 | 80 | 104 | 95 | 83 |
| 1985 | 104 | 112 | 102 | 84 |

| | | | | |
|---|---|---|---|---|
| Total | 104 | 660 | 600 | 506 |
| $A_i$ | 90 | 110 | 100 | 84.33 |
| $\frac{A_i}{G} \times 100$ | 93.67 | 114.49 | 104.07 | 87.77 |

We have $G = \frac{\sum A_i}{4} = \frac{384.33}{4} = 96.08$. Further, since the sum of terms in the last row of the table is 400, no adjustment is needed. These terms are the seasonal indices of respective quarters.

**Merits and Demerits**

This is a simple method of measuring seasonal variations which is based on the unrealistic assumption that the trend and cyclical variations are absent from the data. However, we shall see later that this method, being a part of the other methods of measuring seasonal variations, is very useful.



**Ratio to Trend Method**

This method is used when cyclical variations are absent from the data, *i.e.* the time series variable *Y* consists of trend, seasonal and random components.

Using symbols, we can write Y = T.S.R

Various steps in the computation of seasonal indices are:

(i) Obtain the trend values for each month or quarter, *etc.* by the method of least squares.

(ii) Divide the original values by the corresponding trend values. This would eliminate trend values from the data. To get figures in percentages, the quotients are multiplied by 100.

Thus, we have $\frac{Y}{T} \times 100 = \frac{T.S.R}{T} \times 100 = S.R.100$

(iii) Finally, the random component is eliminated by the method of simple averages.

**Example**

Assuming that the trend is linear, calculate seasonal indices by the ratio to moving average method from the following data:

**Quarterly output of coal in 4 years (in thousand tonnes)**

| *Year* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| 1982 | 65 | 58 | 56 | 61 |
| 1983 | 68 | 63 | 63 | 67 |
| 1984 | 70 | 59 | 56 | 52 |
| 1985 | 60 | 55 | 51 | 58 |

**Solution**

By adding the values of all the quarters of a year, we can obtain annual output for each of the four years. Fit a linear trend to the data and obtain trend values for each quarter.

| *Year* | *Output* | $X$=2($t$-1983.5) | *XY* | $X^2$ |
|---|---|---|---|---|
| 1982 | 240 | -3 | -720 | 9 |
| 1983 | 261 | -1 | -261 | 1 |
| 1984 | 237 | 1 | 237 | 1 |
| 1985 | 224 | 3 | 672 | 9 |
| *Total* | 962 | 0 | -72 | 20 |



From the above table, we get $a = \frac{962}{4} = 240.5$ and $b = \frac{-72}{20} = -3.6$

Thus, the trend line is Y=240.5 – 3.6X, Origin: Ist January 1984, unit of X:6 months.

The quarterly trend equation is given by

$Y = \frac{240.5}{4} - \frac{3.6}{8}$X or Y = 60.13-0.45X, Origin: Ist January 1984, unit of X:1 quarter (*i.e.*, 3 months).

Shifting origin to 15th Feb. 1984, we get

$Y = 60.13 - 0.45(X + \frac{1}{2}) = 59.9 - 0.45X$, origin I-quarter, unit of X=1 quarter.

The table of quarterly values is given by

| *Year* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| 1982 | 63.50 | 63.05 | 62.50 | 62.15 |
| 1983 | 61.70 | 61.25 | 60.80 | 60.35 |
| 1984 | 59.90 | 59.45 | 59.00 | 58.55 |
| 1985 | 58.10 | 57.65 | 57.20 | 56.75 |

The table of Ratio to Trend Values, *i.e.* $\frac{Y}{T} \times 100$

| *Year* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| 1982 | 102.36 | 91.99 | 89.46 | 98.15 |
| 1983 | 110.21 | 102.86 | 103.62 | 111.02 |
| 1984 | 116.86 | 99.24 | 94.92 | 88.81 |
| 1985 | 103.27 | 95.40 | 89.16 | 102.20 |
| Total | 432.70 | 389.49 | 377.16 | 400.18 |
| Average | 108.18 | 97.37 | 94.29 | 100.05 |
| *S.I.* | 108.20 | 97.40 | 94.32 | 100.08 |



Note : Grand Average, $G = \frac{399.89}{4} = 99.97$

**Example** .

Find seasonal variations by the ratio to trend method, from the following data:

| *Year* | *I-Qr* | *II-Qr* | *III-Qr* | *IV-Qr* |
|---|---|---|---|---|
| 1995 | 30 | 40 | 36 | 34 |
| 1996 | 34 | 52 | 50 | 44 |
| 1997 | 40 | 58 | 54 | 48 |
| 1998 | 54 | 76 | 68 | 62 |
| 1999 | 80 | 92 | 86 | 82 |

**Solution**

First we fit a linear trend to the annual totals.

| *Year* | *Annual Totals (Y)* | *X* | *XY* | $X^2$ |
|---|---|---|---|---|
| 1995 | 140 | -2 | -280 | 4 |
| 1996 | 180 | -1 | -180 | 1 |
| 1997 | 200 | 0 | 0 | 0 |
| 1998 | 260 | 1 | 260 | 1 |
| 1999 | 340 | 2 | 680 | 4 |
| Total | 1120 | 0 | 480 | 10 |

Now a = $\frac{1120}{5} = 224$ and $b$ = $\frac{480}{10} = 48$

$\therefore$ Trend equation is Y = 224+48X, origin: Ist July 1997, unit of X = 1 year

The quarterly trend equation is Y=$\frac{224}{4}+\frac{48}{16}$X=56+3X, origin: Ist July 1997, unit of X = 1 quarter.

Shifting the origin to III quarter of 1997, we get

Y = 56 + 3 (X+$\frac{1}{2}$) = 57.5 + 3X



**Table of Quarterly Trend Values**

| *Year* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| 1995 | 27.5 | 30.5 | 33.5 | 36.5 |
| 1996 | 39.5 | 42.5 | 45.5 | 48.5 |
| 1997 | 51.5 | 54.5 | 57.5 | 60.5 |
| 1998 | 63.5 | 66.5 | 69.5 | 72.5 |
| 1999 | 75.5 | 78.5 | 81.5 | 84.5 |

**Ratio to Trend Values**

| *Year* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| 1995 | 109.1 | 131.1 | 107.5 | 93.2 |
| 1996 | 86.1 | 122.4 | 109.9 | 90.7 |
| 1997 | 77.7 | 106.4 | 93.9 | 79.3 |

| | | | | |
|---|---|---|---|---|
| 1998 | 85.0 | 114.3 | 97.8 | 85.5 |
| 1999 | 106.0 | 117.2 | 105.5 | 97.0 |
| Total | 463.9 | 591.4 | 514.6 | 445.7 |
| $A_t$ | 92.78 | 118.28 | 102.92 | 89.14 |
| *S.I.* | 92.10 | 117.35 | 102.11 | 88.44 |

Note that the Grand Average G= $\frac{403.12}{4} = 100.78.$ Also check that the sum of indices is 400.

**Remarks:** If instead of multiplicative model we have an additive model, then Y = T + S + R or *S* + *R* = Y-T. Thus, the trend values are to be subtracted from the Y values. Random component is then eliminated by the method of simple averages.



**Merits and Demerits**

It is an objective method of measuring seasonal variations. However, it is very complicated and doesn't work if cyclical variations are present.

**Ratio to Moving Average Method**

The ratio to moving average is the most commonly used method of measuring seasonal variations. This method assumes the presence of all the four components of a time series. Various steps in the computation of seasonal indices are as follows:

(i) Compute the moving averages with period equal to the period of seasonal variations. This would eliminate the seasonal component and minimise the effect of random component. The resulting moving averages would consist of trend, cyclical and random components.

(ii) The original values, for each quarter (or month) are divided by the respective moving average figures and the ratio is expressed as a percentage, *i.e.* $\frac{Y}{M.A.} = \frac{TCSR}{TCR'} = SR''$, where R´ and R´´ denote the changed random components.

(iii) Finally, the random component R´´ is eliminated by the method of simple averages.

**Example**

Given the following quarterly sale figures, in thousand of rupees, for the year 1996-1999, find the specific seasonal indices by the method of moving averages.

| *Year* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| 1996 | 34 | 33 | 34 | 37 |
| 1997 | 37 | 35 | 37 | 39 |
| 1998 | 39 | 37 | 38 | 40 |
| 1999 | 42 | 41 | 42 | 44 |



**Solution**

**Calculation of Ratio of Moving Averages**

| *Year/ Quarter* | | *Sales* | *4-Period Moving Total* | *Centred Total* | *4 Period M* | $\frac{Y}{M}\times 100$ |
|---|---|---|---|---|---|---|
| 1996 | I | 34 | | … | … | … |
| | II | 33 | | … | … | … |
| | III | 34 | 138 | 279 | 34.9 | 97.4 |
| | IV | 37 | 141 | 284 | 35.5 | 104.2 |
| 1997 | I | 37 | 143 | 289 | 36.1 | 102.5 |
| | II | 35 | 146 | 294 | 36.8 | 95.1 |
| | III | 37 | 148 | 298 | 37.3 | 99.2 |
| | IV | 39 | 150 | 302 | 37.8 | 103.2 |
| 1998 | I | 39 | 152 | 305 | 38.1 | 102.4 |
| | II | 37 | 153 | 307 | 38.4 | 96.4 |
| | III | 38 | 157 | 311 | 38.9 | 97.7 |
| | IV | 40 | 161 | 318 | 39.8 | 100.5 |
| 1999 | I | 42 | 165 | 326 | 40.8 | 102.9 |
| | II | 41 | 169 | 334 | 41.8 | 98.1 |
| | III | 42 | | … | … | … |
| | IV | 44 | | … | … | … |

Calculation of Seasonal Indices

| *Year* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| 1996 | - | - | 97.4 | 104.2 |
| 1997 | 102.5 | 95.1 | 99.2 | 103.2 |
| 1998 | 102.4 | 96.4 | 97.7 | 100.5 |
| 1999 | 102.9 | 98.1 | - | - |
| Total | 307.8 | 289.6 | 294.3 | 307.9 |
| $A_t$ | 102.6 | 96.5 | 98.1 | 102.6 |
| *S.I.* | 102.7 | 96.5 | 98.1 | 102.7 |

Note that the Grand Average $G=\frac{399.8}{4}=99.95$. Also check that the sum of indices is 400.

**Merits and Demerits**

This method assumes that all the four components of a time series are present and, therefore, widely used for measuring seasonal variations. However, the seasonal variations are not completely eliminated if the cycles of these variations are not of regular nature. Further, some information is always lost at the ends of the time series.



**Line Relatives Method**

This method is based on the assumption that the trend is linear and cyclical variations are of uniform pattern. As discussed in earlier chapter, the link relatives are percentages of the current period (quarter or month) as compared with previous period. With the computation of link relatives and their average, the effect of cyclical and random component is minimised. Further, the trend gets eliminated in the process of adjustment of chained relatives. The following steps are involved in the computation of seasonal indices by this method:

(i) Compute the link relative (*L.R.*) of each period by dividing the figure of that period with the figure of previous period. For example, link relative of

$$3^{rd} \text{ quarter} = \frac{\text{figure of } 3^{rd} \text{ quarter}}{\text{figure of } 2^{nd} \text{ quarter}} \times 100$$

(ii) Obtain the average of link relatives of a given quarter (or month) of various years. *A.M.* or $M_d$ can be used for this purpose. Theoretically, the later is preferable because the former gives undue importance to extreme items.

(iii) These averages are converted into chained relatives by assuming the chained relative of the first quarter (or month) equal to 100. The chained relative (*C.R.*) for the current period (quarter or month)

$$= \frac{C.R. \text{ of the previous period} \times L.R. \text{ of the current period}}{100}$$

(iv) Compute the C.R. of first quarter (or month) on the basis of the last quarter (or month). This is given by

$$= \frac{C.R. \text{ of the last quarter (or month)} \times L.R. \text{ of } 1^{st} \text{ quarter (or month)}}{100}$$

This value, in general, be different from 100 due to long term trend in the data. The chained relatives, obtained above, are to be adjusted for the effect of this trend. The adjustment factor is

$d = \frac{1}{4}$ [New *C.R.* for Ist quarter-100] for quarterly data

and $d = \frac{1}{12}$ [New C.R. for Ist month –100] for monthly data.

On the assumption that the trend is linear, $d$, $2d$, $3d$, etc. is respectively subtracted from the $2^{nd}$, $3^{rd}$, $4^{th}$, *etc.,* quarter (or month).

(v) Express the adjusted chained relatives as a percentage of their average to obtain seasonal indices.

(vi) Make sure that the sum of these indices is 400 for quarterly data and 1200 for monthly data.

**Exercise**

1. What effect does seasonal variability have on a time-series? What is the basis for this variability for an economic time-series?
2. What is measured by a moving average? Why are 4-quarter and 12- month moving averages used to develop a seasonal index?
3. Briefly describe the moving average and least squares methods of measuring trend in time-series.
4. Distinguish between ratio-to-trend and ratio-to-moving average as methods of measuring seasonal variations, which is better and why?
5. Why do we deseasonalize data? Explain the ratio-to-moving average method to compute the seasonal index.
6. For what purpose do we apply time series analysis to data collected over a period of time?
7. What is the difference between a causal model and a time series model?
8. Explain clearly the different components into which a time series may be analysed. Explain any method for isolating trend values in a time series.
9. Explain what you understand by time series. Why is time-series considered to be an effective tool of forecasting?
10. The sales (Rs. In lakh) of a company for the years 1990 to 1996 are given below:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Year : | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 |
| Sales : | 32 | 47 | 65 | 88 | 132 | 190 | 275 |

Find trend values by using the equation $Y_c = ab^x$ and estimate the value for

1997.