Chapter
8

Solved Problems on Numerical Technique

Problem: Find the Picard approximations y_1,y_2, y_3 to the solution of the initial value problem $y'=y$, $y(0)=2$. Use y_3 to estimate the value of y (0.8) and compare it with the exact solution.

Solution: Let $y_0=2$, the value of y_1 is $y_1=2+\int_0^x 2dt = 2+2x$

$y_2 = 2+\int_0^x (2+2t)dt=2+2x+x^2$

$y_3=2+\int_0^x (2+2t+t^2)dt = 2+2x+x^2+\frac{1}{3}x^3$

At x= 0.8, $y_3=2+2(0.8)+(0.8)^2+\frac{1}{3}(0.8)^3=4.41$

The solution of the initial-value problem, found by separation of variables, is $y = 2e^x$. At x = 0.8, $y=2e^{0.8} = 4.45$.

Problem: A certain chemical reaction takes place such that the time-rate of change of the amount of the unconverted substance q is equal to -2q. If the initial mass is 50 grams, use the Runge-Kutta method to estimate the amount of unconverted substance at t = 0.8 sec.

Solution: The initial-value problem is $\frac{dq}{dt}$ = -2q, q(0) = 50

Using h = 0.8 in the Runge-Kutta formulas,

m_1 = -2(50) = -100

m_2 = -2[50 + 0.8(-100)/2] = -20

m_3 = -2[50 + 0.8(-20)/2] = -84

m_4 = -2[50 + 0.8(-84)] = 34.4

Therefore our estimate of the mass of unconverted substance at t = 0.8 is q(0.8) = 50 + $\frac{1}{6}$(0.8) [-100 + 2(-20) + 2(-84) + 34.4] = 13.5 g

Problem: Use the Runge-Kutta method to estimate y (0.4) if y' = 2x + y, y (0) = 1

Solution: In using the Runge - Kutta formulas, we note that f(x, y) = 2x + y, x_0 = 0, and y_0 = 1. Choosing h = 0.4, we have

m_1 = [2(0) + 1] = 1.0

m_2 = [2(0 + 0.4/2) + (1 + 0.4(1.0)/2)] = 1.6

m_3 = [2(0 + 0.4/2) + (1 + 0.4(1.6)/2)] = 1.72

m_4 = [2(0 + 0.4) + (1 + 0.4(1.72))] = 2.488

Hence

$$y\,(0.4) = 1 + \frac{1}{6}(0.4)[1.0 + 2(1.6) + 2(1.72) + 2.488] = 1.675$$

Problem: Use the improved Euler method with h = 0.4 to estimate y (0.4) if y'= 2x + y, y (0) =1

Solution: By the Euler method with h=0.4 we obtain an estimate for y (0.4). By equation $y_1 = y_0 + h\, f(x_0, y_0)$ for $x_0 = 0$, $y_0 = 1$, and h= - 0.4

We get $y_1 = 1.0 + 0.4\,[2(0) + 1.0] = 1.4$

Thus $y_1 = 1.4$ is the approximate value.

This corresponds to y_t in the improved Euler method. Therefore

$$M = \frac{1}{2}[f(0,1)+f(0.4,1.4)] = \frac{1}{2}[2(0)+1)+(2(0.4)+1.4)] = 1.6$$

The value of M is now used in $y_1 = y_0 + hM$. Thus $y_1 = 1 + 0.4(1.6) = 1.64$ is the estimate of y (0.4).

Exercise

1. Write sufficient condition for convergence of an iterative method for $f(x) = 0$.

2. Write down the procedure to find the numerically smallest eigen value of a matrix by power method.

3. Form the divided difference table for the data **(0,1) , (1, 4) , (3, 40)** and **(4, 85)** .

4. Define a cubic spline $S(x)$ which is commonly used for interpolation.

5. State the Romberg's integration formula with h_1 and h_2. Further, obtain the formula when $h_1 = h$ and $h_2 = h/2$.

6. Use Euler's method to find $y(0.2)$ and $y(0.4)$ given $\frac{dy}{dx} = x + y$,, $y(0) = 1$.

7. Write the Adam – Bashforth predictor and corrector formulae.

8. Write down the explicit finite difference method for solving one dimensional wave equation.

9. Write down the standard five point formula to find the numerical solution of Laplace equation.

10. Solve for a positive root of the equation $x^4 - x - 10 = 0$

    using Newton – Raphson method.

Exercise

11. Use Gauss – Seidal iterative method to obtain the solution of the equations:
$9x - y + 2z = 9$; $x + 10y - 2z = 15$; $2x - 2y - 13z = -17$
.

12. Use Lagrange's formula to find a polynomial which takes the values **$f(0) = -12$, $f(1) = 0$, $f(3) = 6$** and **$f(4) = 12$** . Hence find **$f(2)$** .

13. Find the function **$f(x)$** from the following table using Newton's divided difference formula:

x :	0	1	2	4	5	7
$f(x)$:	0	0	–12	0	600	7308

14. The velocity u of a particle at a distance S from a point on its path is given by the table below:

S (meter)	0	10	20	30	40	50	60
u (m / sec)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by Simpson's $1/3^{rd}$ rule and Simpson's $3/8^{th}$ rule.

15. Given the data f(1) = 4, f(2) = 5, f(7) =5, f(8)= 4. Compute. f(6) using Lagrange' interpolation formula.

Exercise

16. Define diagonally dominant system of equations. Solve the following system of equations using Gauss-Seidal method :

10x + 15y + 3z= 14 -30x+y+5z=17 © xt+y+4z=3.,

17. Use Tailor series method to find ***y*(0.1)** and ***y*(0.2)** given that $\frac{dy}{dx}$ **= 3 *e*^{x} + 2 *y*,*y*(0) = 0** , correct to 4 decimal accuracy.

18. Using Newton-Raphson method, find a root correct to three decimal places of the equation sin x = 1 – x.

19. Find a root of the equation x3 - x - 11 = 0 correct to four decimals using bisection method.

20. Using Newton-Raphson method, find a root correct to three decimal places of the equation x3- 3x - 5 = 0.

21. Which of the following statements applies to the bisection method used for finding roots of functions?

A.Converges within a few iterations

B.Guaranteed to work for all continuous functions (Ans)

C.Is faster than the Newton-Raphson method

D.Requires that there be no error in determining the sign of the function

Exercise

22. In the Gauss elimination method for solving a system of linear algebraic equations,triangularzation leads to

A.Diagonal matrix

B.Lower triangular matrix (Ans)

C.Upper triangular matrix

D. Singular matrix

23. Which of the following statements applies to the bisection method used for finding roots of functions?

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C. Is faster than the Newton-Raphson method

D. Requires that there be no error in determining the sign of the function

24. The convergence of which of the following method is sensitive to starting value?

A.False position

B.Gauss seidal method

C.Newton-Raphson method

D.All of these

25. If $\Delta f(x) = f(x+h) - f(x)$, then a constant k, Δk equals

A. 1 B. 0 c.f(k)-f(0) D.f(x+k) –f(x)