$1 + G(s)H(s) = 0$

$$1 + \frac{1}{s(s+1)} = 0$$

$s(s+1) + 1 = 0$

$s^2 + s + 1 = 0$

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\omega_n = 1, \quad 2\xi\omega_n = 1$

$2\xi = 1$

$\xi = 1/2 = 0.5$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$= e^{-\pi \times \frac{1}{2}/\sqrt{1-1/4}} = e^{\frac{-\pi/2}{\sqrt{3/4}}} = 0.163$$

M_p = Max overshoot − e_{ss}

Delay-time :- It is time required for response to reach 50% of final value in first attempt.

$$t_d = \frac{1 + 0.7\xi}{\omega_n} \quad \boxed{0 < \xi < 1.0}$$

$$\left.\begin{aligned} t_d &\propto \xi \\ t_d &\propto \frac{1}{\omega_n} \end{aligned}\right\}$$

Rise-time :-

0 − 100% of its final value for under damped.

10% − 90% of its final value for over-damped.

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

for calculation of Rise time simply put

$y(t) = 1$

$\sin(\omega_d t_r + \phi) = 0$

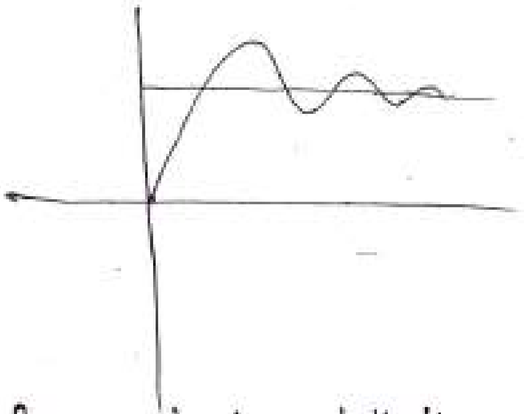
$\omega_d t_r + \phi = \pi$

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

$$t_r = \frac{0.8 + 2.5\xi}{\omega_n}$$

$0 < \xi < 1.$

Settling time → It is time required by system to reach & stay within a specified tolerance band (2% or 5%) of its final value.



for 2% band it takes 4 time const.

$$t_s = \frac{4}{\xi \omega_n}$$

for 5% band it takes 3 time const.

$$t_s = 3/\xi\omega_n$$

These formula are not valid for over-damped.

for over-damped

$$t_s = \frac{2\xi}{\omega_n} \text{ for 2\% band}$$

* Q. GATE 2002

The T/F fn. of a system is

$$\frac{G(s)}{} = \frac{100}{(s+1)(s+100)}$$

for a unit step i/p to system the approx. settling time for 2% criteria is

a) 100 sec.

b) 4 sec. ✓

c) 1 sec. (+)

D) 0.01 sec.

Q./GATE For a II^{nd} order system with C.L.T.F.

$$T(s) = \frac{9}{s^2+4s+9}$$. the settling time for 2% band in sec. is

A) 1.5 B) 2 (✓)

C) 3 D) 4

Sol^n

$$t_s \; s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = 3$$

$$2\zeta\omega_n = 4$$

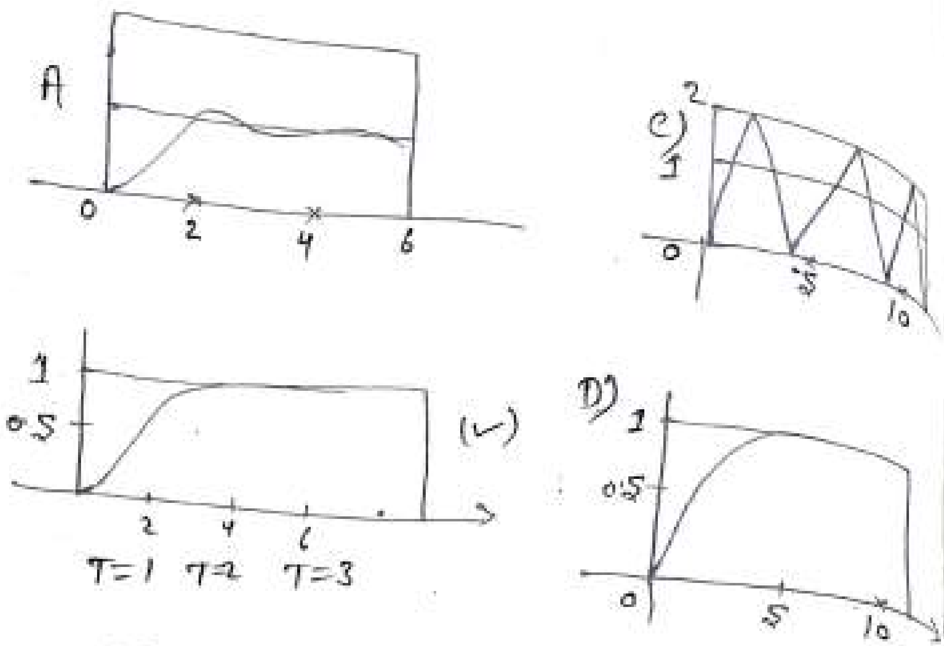
$$2 \times 3 \times \zeta = 4$$

$$\zeta = 4/6 = 2/3$$

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{2} = 2$$

Q. A sec. order system has transfer f^n.

$$\frac{C(s)}{R(s)} = \frac{4}{s^2+4s+4}$$ with r(t) as unit step i/p. what is response c(t).



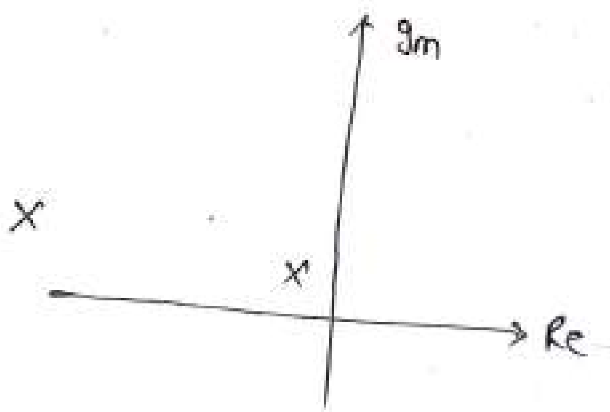
$2\xi\omega_n = 4 \quad \omega_n = 2$

$\xi = 1 \Rightarrow$ critical damped

$\xi\omega_n = 2$ → Time period.

Dominant pole & Insignificant pole:-

Dominant pole are poles which are closer to imaginary axis in left half of s-plane give rise to transient response. These Dominant pole are used to



control dynamic performance of system.

Insignificant poles → If dis of pole 5 to 10 times of dominant pole then this pole may be regarded as insignificant pole as far as transient ~~poles~~ response is concerned.

these insignificant pole are used for purpose of ensuring that controller transfer f^n can be realized by physical component. (11)

Q. calculate damping ratio of given system

$$\frac{C(s)}{R(s)} = \frac{20}{(s+10)(s^2+2s+2)}$$

Soln

$$= \frac{20}{10\underbrace{(1+s/10)}(s^2+2s+2)}$$

$$= \frac{2}{s^2+2s+2}$$

$\omega_n = \sqrt{2}$

$2\xi\omega_n = 2$

$\xi = 1/\sqrt{2}$.

Type & order of system →

~~$G(s)H(s) = \frac{K(1+sT_a)}{}$~~

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)(1+sT_c)----}{s^n(1+sT_1)(1+sT_2)----}$$

No. of pole at origin. decides Type of system

Q. $G(s)H(s) = \frac{K(1+s)}{s^2(1+3s)}$

Type → 2
order → 3

Q. $G(s)H(s) = \frac{1}{s(1+s)}$

Type → 1
order → 2

Time response of unit Impulse IInd order System:-

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \qquad \text{If } r(t) = \delta(t) \quad R(s) = 1$$

$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\boxed{c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t)}$$

In case I:- $\xi < 1$

$$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t)$$

case II:- If $\xi = 1$

$$c(t) = t\omega_n^2 e^{-\xi\omega_n t}$$

case III, If $\xi > 1$

$$\frac{\omega_n}{2\sqrt{\xi^2-1}} \left[e^{-(\xi+\sqrt{\xi^2-1})\omega_n t} - e^{-(\xi-\sqrt{\xi^2-1})\omega_n t} \right]$$

Steady-state Errors →

Q. If Gain is (↑) then Steady state Error is

A) ↑ B) ↓ (✓)

C) Remain unchanged D) None

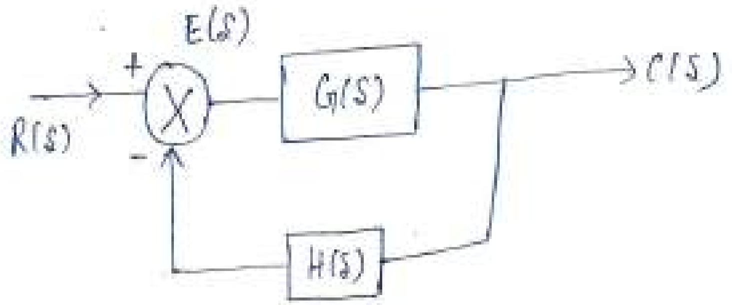
Sol.n By increasing the gain damping will decrease this implies response will be faster this employ S.S.E decrease.

gain(↑) → damping (↓) → fast response ⇒ S.S.E (↓)

Static

Error coeff. $\Rightarrow$ S.S.E should be min for a better control system.

1) Static position Error coeff. (K_p)
2) Static vel. Error coeff. (K_v)
3) Static accele.- Error coeff. (K_a)



$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$e(t) = r(t) - c(t)$$

$$e(t) = \text{Error}$$

Steady-state value of $e(t)$

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$\Rightarrow \boxed{\text{S.S.E } (e_{ss}) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)}}$$

1. (K_p):- this is for unit step i/p

$$r(t) = u(t) \Rightarrow R(s) = 1/s$$

②

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{s \times 1/s}{1+G(s)H(s)} = \lim_{s\to 0} \frac{1}{1+G(s)H(s)}$$

S.S.E for unit step i/p is

$$e_{ss} = \frac{1}{1+\lim_{s\to 0} G(s)H(s)} = \frac{1}{1+K_p}$$ where

$K_p = \lim_{s\to 0} G(s)H(s)$.

2) for unit Ramp (steady-state Error)

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$r(t) = t$
$R(s) = \frac{1}{s^2}$

$$= \lim_{s\to 0} \frac{s \times 1/s^2}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{1}{s+s\,G(s)H(s)}$$

$$= \frac{1}{0+\lim_{s\to 0} s\,G(s)H(s)} = \frac{1}{K_v}$$

$$e_{ss} = \frac{1}{K_v}$$

where $K_v = \lim_{s\to 0} sG(s)H(s)$

for unit ramp $e_{ss} = \frac{1}{K_v}$

Case 3 - for Parabolic

$$r(t) = \frac{t^2}{2}$$

$$e_{ss} = \frac{1}{K_a}$$

where $K_a = \lim_{s\to 0} s^2 G(s)H(s)$

Q. What is e_{ss} for Type 0 system with unit step as i/p ??

Solⁿ

$\cancel{G(s)H(s)=0}$

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\ldots}{s^N(1+sT_1)(1+sT_2)}$$

$$e_{ss} = \frac{1}{1+K_p}$$

for Type 0

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\ldots}{(1+sT_1)(1+sT_2)\ldots}$$

$$= K$$

$$e_{ss} = \frac{1}{1+K}$$

Q. What is e_{ss} for Type 0 system with unit Ramp as i/p.

Solⁿ

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} s\,G(s)H(s)$$

$$= \lim_{s\to 0} s \cdot \frac{K(1+sT_a)(1+sT_b)\ldots}{(1+sT_1)(1+sT_2)\ldots}$$

$$= 0$$

Similarly $e_{ss} = \infty$ for Type 0 with unit parabolic as i/p

Calculate e_{ss} for Type 2, Type 1 with all 3 i/p's.

Solⁿ for Type 1:-

(i) unit step:-

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$= \frac{K(1+sT_a)(1+sT_b)\ldots}{s(1+sT_1)(1+sT_2)\ldots}$$

$$= \infty$$

$$e_{ss} = \frac{1}{1+K_p} = 0$$

(ii) unit Ramp:-

$$K_v = \lim_{s\to 0} s\,G(s)H(s)$$

$$= \lim_{s\to 0} \cancel{s}\cdot\frac{K(1+sT_a)(1+sT_b)\cdots}{\cancel{s}(1+sT_1)(1+sT_2)\cdots}$$

$$= K$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}$$

(iii) unit parabolic:-

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$$= \lim_{s\to 0} s^2\cdot\frac{K(1+sT_a)(1+sT_b)\cdots}{s(1+sT_1)(1+sT_2)\cdots}$$

$$= 0$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

for type 2:-

(i) unit step:-

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$= \lim_{s\to 0}\frac{K(1+sT_a)(1+sT_b)\cdots}{s^2(1+sT_1)(1+sT_2)\cdots}$$

$$= \infty$$

$$e_{ss} = \frac{1}{1+K_p} = 0$$

(ii) unit Ramp:-

$$K_v = \lim_{s\to 0} s\,G(s)H(s)$$

$$= \lim_{s\to 0} s\cdot\frac{K(1+sT_a)(1+sT_b)\cdots}{s^2(1+sT_1)(1+sT_2)\cdots}$$

$$= \infty$$

$$e_{ss} = \frac{1}{K_v} = 0$$

(iii) unit parabolic

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$$= \lim_{s\to 0} s^2\cdot\frac{K(1+sT_a)(1+sT_b)\cdots}{s^2(1+sT_1)(1+sT_2)\cdots}$$

$$= K$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K}$$

g/p	Type 0 (ess)	Type 1 (ess)	Type 2 (ess)
1. unit step	$\frac{1}{1+K}$	0	0
2. unit Ramp	∞	$\frac{1}{K}$	0
3. unit parabolic	∞	∞	$\frac{1}{K}$

Note:
So, It is clear from Table that unit ramp & unit parabolic are not acceptable for Type 0 while unit ~~ramp~~ parabolic is not acceptable for Type 1 system

Q. IES-05 what is steady state Error for a unity feedback control system having $G(S) = \frac{1}{S(S+1)}$ due to unit ramp. g/p ?

A) 1

B) 0.5

C) 0.25

D) $\sqrt{0.5}$

Soln
for unit ramp, $e_{ss} = \frac{1}{K_v}$

$$K_v = \lim_{S\to 0} S\,G(S)\,H(S)$$

$$= \lim_{S\to 0} \frac{S}{S(S+1)} = 1$$