01/11/07

Signal / system

GATE :- (15-20) Mark's

IES :- 20 Marks Q'S

1. conventional q's in paper-I → (40)

→ Random variable → (4-5 Marks)

Q. what is Signal ?

Q. " " System ?

$$\text{I/P} \rightarrow \boxed{DSP} \rightarrow \text{O/P}$$

→ System

Signal is the physical quantity that varies with time & space or other independent variable so it is nothing but f'n of one or more independent variable.

$S = 10t + 2t^2$ → It is Signal which is f'n of time.

$s(x,y) = 3xy + 5xy^2$ → It is Signal which is f'n of x & y.

But there are classes of signals which are not f'n of independent variable e.g. speech.

System:- It is a device which operate on signal is called System.

Ex. DSP, ASP

1. Continuous
2. Discrete
3. Analog.
4. Digital

1. Independent variable
2. Dependent variable.

$y = 2t + 3t^2$

↳ Independent variable

↳ Dependent variable.

Continuous

1) ∞

Discrete

finite → Independent variable

Analog

∞

Digital

Finite — Dependent. Variable

Analog →

1:02000
1:02001
1:02002
,
,
,
1:02099

1:02

↓

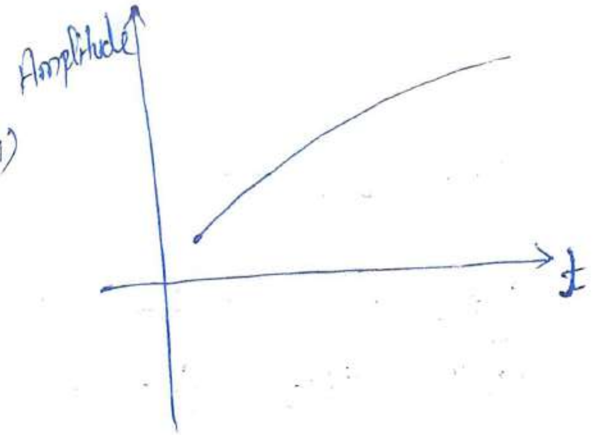
1:03

← Digital.

Types of Signal —

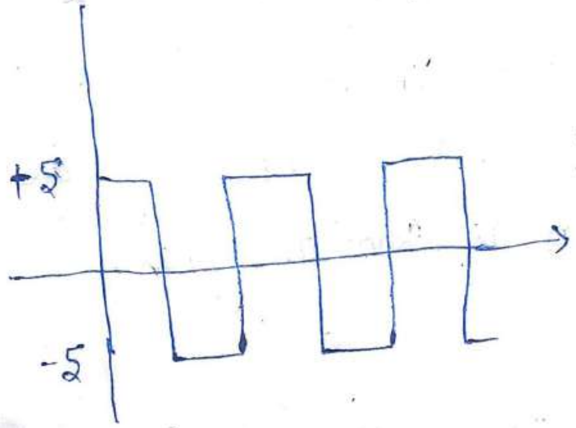
1) Continuous Analog
2) " Digital
3) Discrete Analog
4) " Digital

1)



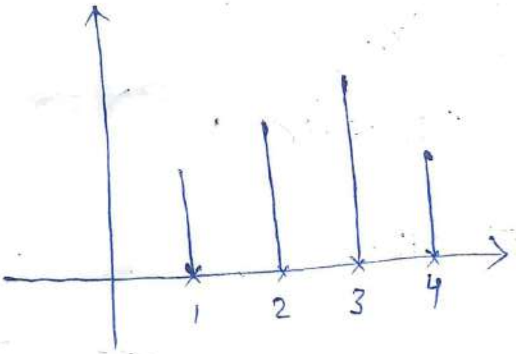
→ Continuous Analog

2)



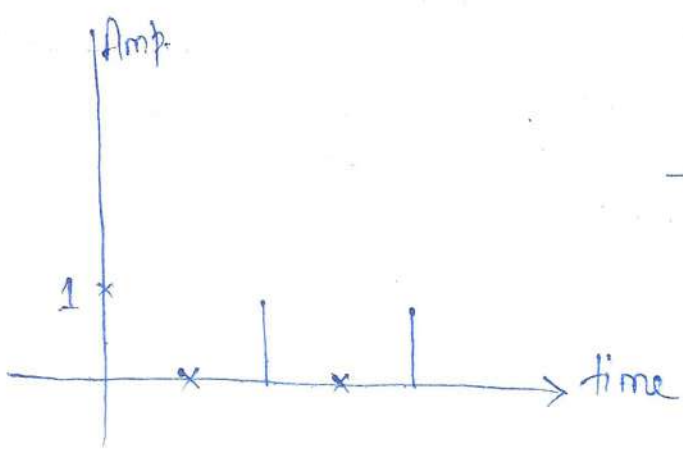
→ C.D.

3)



→ D.A

4)



→ D.D

Binary signal is Ex. of Discrete Digital

Types of Signal:-

1. Deterministic & Random Signal $\rightarrow$

Any signal that can be uniquely describe by an Exclusive Mathemathical Expression is known as Deterministic signal. Here past, present & future value are known without any uncertainty.

○ The signal which ~~cannot be~~ cannot be describe explicity, is known as Random signal.
स्पष्ट रूप से

Q. Random signal can be modelled by

a) differential Equ.
b) Statistical parameter. (✓)
c) Difference Equ.
d) Integral ,,

Energy of a signal:-

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

power of a signal —

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Energy Signal & power Signal →

For a power signal

P_s = Finite

E_s = Infinite

for Energy Signal

E_s = finite

$P_s = 0$

Q. Energy of a signal Means:—

IES.

1. That much Energy can Extract from signal

2. Actual Energy of signal

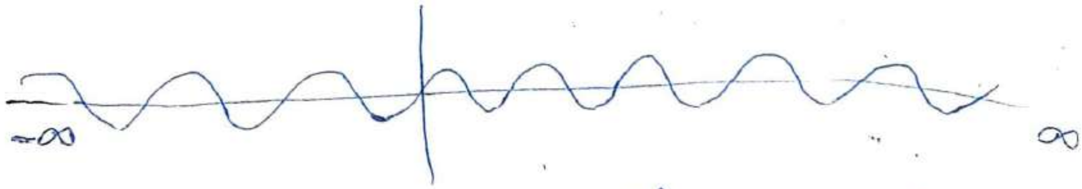
3. Energy capab. of. a signal

A) 1,2 c) 1,3 (✓)

B) 2,3 d) 1,2,3

So, Energy of signal means this much Energy can be Extract from signal & these are not actual Energy of signal. Generally Energy & power calculation are done for load of 1 Ω resistance i.e value of Energy & power in 1Ω resistor.

All periodic signals are power signal.



So, theoritically it is not possible to generate power sign in lab.

Q. What is power of signal

$$x(t) = C\cos(\omega_0 t + \theta)$$

$$P = \lim_{t \to \infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} x^2(t)\,dt.$$

$$= \lim_{t \to \infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} C^2\cos^2(\omega_0 t + \theta)\,dt.$$

$$\boxed{P = \frac{C^2}{2}}$$

Note:- for sinusoidal signal power will be simply $\frac{A_m^2}{2}$

$x(t) = A_m \cos\omega t.$

Q. What is power of signal

$$x(t) = C_1\cos(\omega_0 t + \theta) + C_2\cos(\omega_0 t + \theta)$$

$$P = \frac{C_1^2 + C_2^2}{2} \rightarrow \text{RMS value} = \sqrt{P}$$

$$= \sqrt{\frac{C_1^2 + C_2^2}{2}}$$

RMS value $= A_m/\sqrt{2}$

Q. GATE-2001 / ES-03 If a signal $f(t)$ has Energy E then energy of signal $f(2t)$ is

a) E b) E/2 (✓)

c) 2E d) 4E

Ans

$$E = \int_{-\infty}^{\infty} f(t)^2\, dt$$

$$E' = \int_{-\infty}^{\infty} f(2t)^2\, dt$$

put $2t = z$

$dt = dz/2$

$$= \int_{-\infty}^{\infty} \frac{f(z)^2\, dz}{2}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} f(z)^2 dz = E/2$$

Note :- $f(t) \rightarrow E.$

$f(at) \rightarrow E/a.$

Q. -2005 / ES-01 The power in signal

$$s(t) = 8\cos(20\pi t - \pi/2) + 4\sin 15\pi t$$

A) 40 (✓) B) 41

c) 42 d) 82

Ans.

$$P = \frac{C_1^2 + C_2^2}{2} = \frac{8^2+4^2}{2} = \frac{64+16}{2} = \frac{80}{2}$$

$= 40$

The signal

$x(t) = A\cos(\omega_0 t + \phi)$ is

A) power signal (✓)

B) Energy "

c) Both Energy & power

d) Neither Energy nor power.

2. What is R.M.S. value of

5-98

$x(t) = A_m e^{j\omega_0 t}$

A) A_m (✓)

B) $A_m/\sqrt{2}$

c) $A_m/2$

d) $2A_m$

$A_m\cos\omega_0 t + A_m\sin\omega_0 t$

$$P = \frac{A_m^2}{2} + \frac{A_m^2}{2} = A_m^2$$

$RMS = A_m$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$$

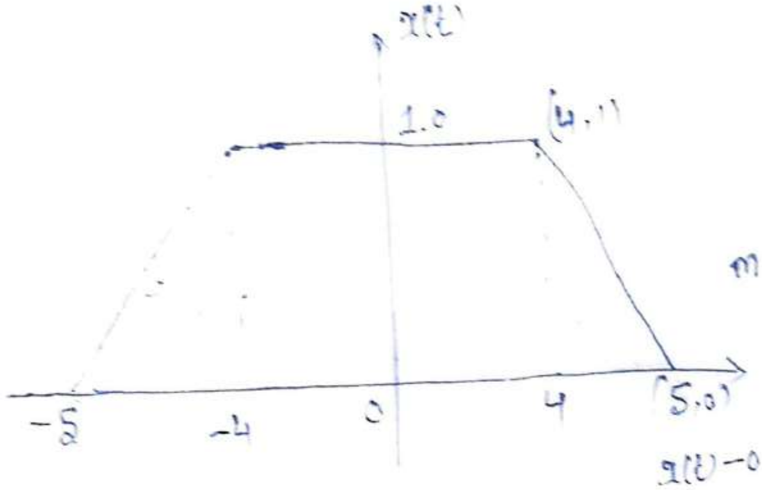
$|e^{j\omega_0 t}| = 1$

$$= \frac{1}{T}\int_{-T/2}^{T/2} A_m^2\,dt$$

$P = A_m^2$

RMS value $= A_m$

Q. calculate energy of given signal



$x(t) - 0 = -1(t-5)$

$x(t) = 5-t$

Sol:

$$x(t) = \begin{cases} 1 & 0<t\le 4 \\ 5-t & 4<t<5 \end{cases}$$

$$E = 2\int_0^4 (1)^2 dt + 2\int_4^5 (5-t)^2 dt$$

$$= 2(4-0) + 2\int_4^5 (25+t^2-10t)\,dt$$

$$= 2(4) + 2\left[25t + \frac{t^3}{3} - \frac{10t^2}{2}\right]_4^5$$

$$= 8 + 2\left[41.6 - 41.33\right]$$

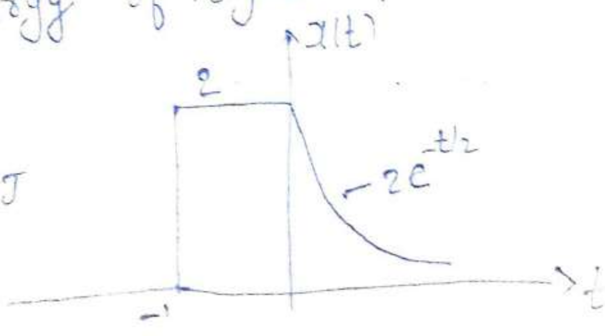
$= 8.54$ joule.

Q. calculate energy of signal ??

A) 4J B) 8J

c) 2J d) 16J

Sol:



Ans.

$$x(t) = \begin{cases} 2 & -1 < t \leq 0 \\ 2e^{-t/2} & 0 < t < \infty \end{cases}$$

$$E = \int_{-1}^{0} (2)^2 dt + \int_{0}^{\infty} (2e^{-t/2})^2 dt$$

$$= 4(1) + 4 \int_{0}^{\infty} e^{-t} dt$$

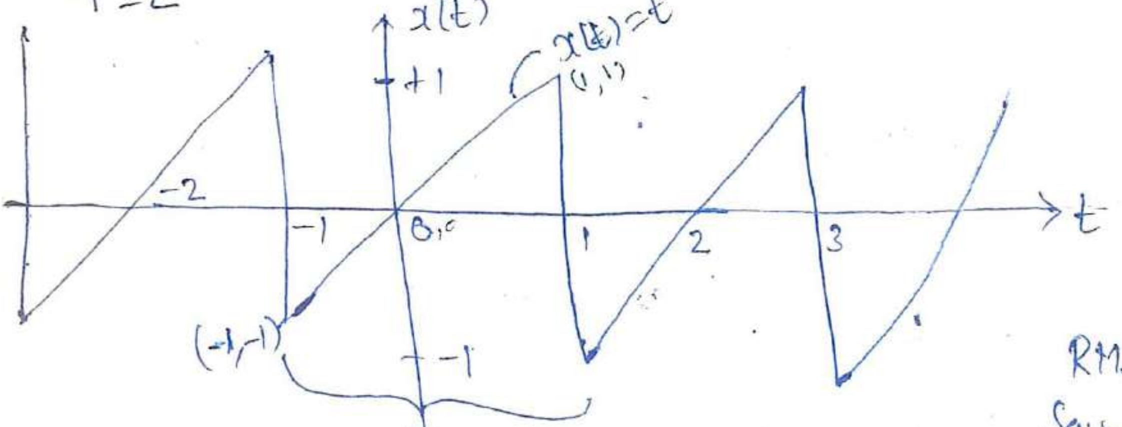
$$= 4 + 4 \cdot \left[-e^{-t} \right]_0^{\infty}$$

$$= 4 + 4 \cdot \left[0 + 1 \right]$$

$$= 8 \text{ J}$$

Q. calculate power of signal?

$T=2$



A) $1/2$ B) $1/3$ (✓)

C) 1 d) $1/4$

RMS value of Saw-tooth is

$$P = \frac{A_m^2}{3}$$

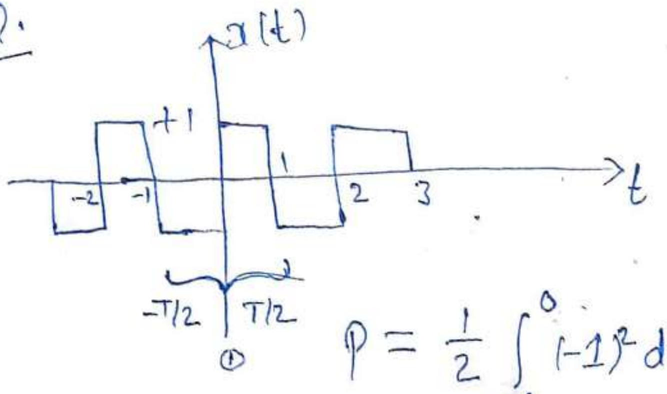
Soln

$$\cancel{P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt.}$$

$T=2$ $$P = \frac{1}{2} \int_{-1}^{1} |x(t)|^2 dt = \frac{1}{2} \int_{-1}^{1} t^2 dt$$

$$= 1/3$$

Q.



$$P = \frac{1}{2} \int_{-1}^{0} (-1)^2 dt + \frac{1}{2} \int_{0}^{1} (1)^2 dt$$

$$= \frac{1}{2} [0+1] + \frac{1}{2} [1-0] = \frac{1}{2} + \frac{1}{2} = 1$$

$P = Am$ $RMS = \sqrt{Am}$

Q. Ramp is

A) An Energy Signal

B) power signal

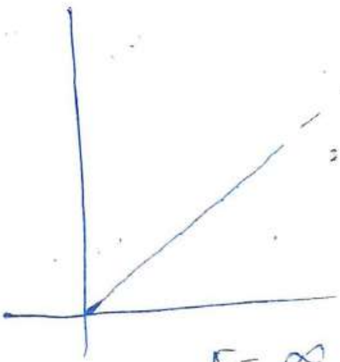
c) Both Energy & power

d) Neither Energy nor power (✓)

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} t^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[\frac{t^3}{3} \right]_{-T/2}^{T/2}$$

$$= \infty$$



$T = 0$

$E = \infty$

$P = \infty$

$P = \frac{1}{T}$

Periodic Signal:-

if $x(t) = x(t + T_0)$
then Signal is called periodic signal.
with Time period T_0.

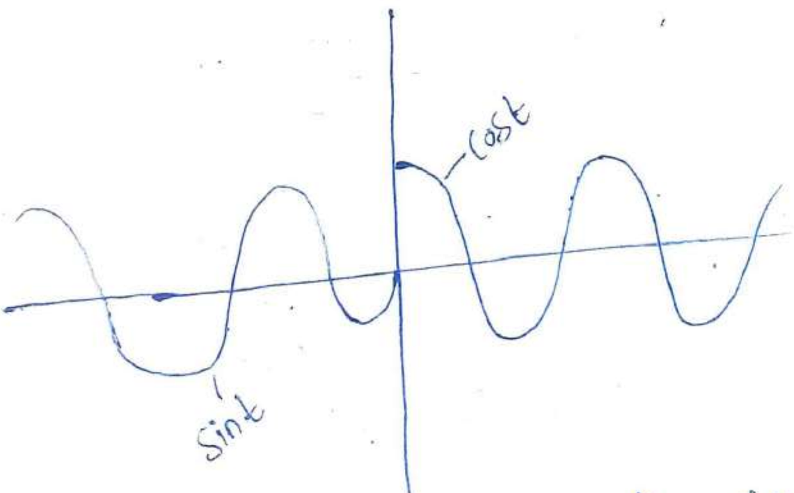
So, Periodic signal $x(t)$ remains unchange when time shifted by period T_0. Periodic signal start from $T = -\infty$ & terminate at $T = +\infty$.

Note:- condition is that there should not be any discontinuity]

Q.

$$x(t) = \begin{cases} \cos t , & t \geq 0 \\ \sin t , & t < 0 \end{cases}$$

Is it periodic signal



So, Signal $x(t)$ is not periodic signal. Becoz here is discontinuity.

Calculation of Time period for Periodic signals

$$x(t) = e^{j\omega_0 t}$$

$$x(t+T_0) = e^{j\omega_0 (t+T_0)}$$

for periodic signal

$$x(t) = x(t+T_0)$$

$$\therefore e^{j\omega_0 t} = e^{j\omega_0 (t+T_0)}$$

$$\therefore e^{j\omega_0 T_0} = 1 = e^{j2\pi}$$

$$T_0 = \frac{2\pi}{\omega_0}$$

Q. $x(t) = 3\cos(5t + 30^\circ)$

Ans. $\omega_0 = 5$ $\quad T = \frac{2\pi}{5}$

Q. $x(t) = \cos(2t + 45^\circ)$

Ans $\omega_0 = 2$ $\quad T = \frac{2\pi}{2} = \pi$

Q. $x(t) = \cos^2 t = \frac{1 + \cos 2t}{2}$

$\omega_0 = 2$

Ans $\quad T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi$

Q. $x(t) = j e^{j\pi t}$

$e^{j\theta} = \cos\theta + j\sin\theta$

$$j = e^{j\pi/2}$$

$$= \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$$

$$= e^{j(\pi t + \pi/2)}$$

$$= e^{j(\omega_0 t + \theta)}$$

$\omega_0 = \pi, \Rightarrow T = \frac{2\pi}{\pi} = 2$

Q. $x_1(t)$ is periodic with time period T_1
$x_2(t)$ T_2

$$x(t) = x_1(t) + x_2(t)$$

$\downarrow$

$x(t)$ is periodic only if $\frac{T_1}{T_2}$ = rational no.

Q. $x(t) = 4\sin 3t + 3\sin\sqrt{3}\,t$

Ans. $\frac{2\pi}{3}$ $\qquad$ $\frac{2\pi}{\sqrt{3}}$

$$\frac{T_1}{T_2} = \frac{\frac{2\pi}{3}}{\frac{2\pi}{\sqrt{3}}} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \text{Irrational no.}$$

$x(t)$ is non periodic.

Q. $x(t) = 10\sin 5t - 4\cos 7t$

$T_1 = \frac{2\pi}{5}, \quad T_2 = \frac{2\pi}{7}$

$\frac{T_1}{T_2} = \frac{7}{5}$ = Rational

If periods $T_1 = \frac{p}{q}$, $T_2 = \frac{R}{S}$

$$T = \frac{\text{LCM of } p \,\&\, R}{\text{HCF of } q \,\&\, S} = \frac{2\pi}{1} = 2\pi$$

$\frac{22}{7} = 3.142857143$ – irrational

$\frac{4}{3} = 1.33333 \ldots$ – rational

Q. $x(t) = \cos 3t + \cos 6\pi t$

$T_1 = \frac{2\pi}{3}, \quad T_2 = \frac{1}{3}$

$\frac{T_1}{T_2} = \frac{2\pi}{1}$ = irrational

a) Periodic with period 2π

b) " " " $2\pi/3$

c) " " " $2\pi/6$

d) Non periodic (✓)

Q. $x(t) = \cos \pi t + 2\cos 3\pi t + 3\cos 5\pi t$

$T = ??$

Ans.

$T_1 = \frac{2\pi}{\pi} = 2$

$T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$

$T_3 = \frac{2\pi}{5\pi} = \frac{2}{5}$

$$T = \frac{\text{LCM of } 2,2,2}{\text{HCF of } 1,3,5} = 2$$

Hence, it period 2.

~~Note: If $x_1(t)$ or $x_2(t)$ does not have π as coeff~~

03/10/07

$$E(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt \rightarrow \text{Cont. signal}, \quad P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 \rightarrow \text{Discrete signal} \quad P = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} [x(n)]^2$$

Q. if $x(n) = u[n]$
What is Energy of signal?

Ans.

$$u(n) = 1 \quad n > 0, \quad = 0 \quad n < 0$$

$$E = \sum_{n=-\infty}^{\infty} |u(n)|^2$$

$$= \sum_{n=0}^{\infty} (1)^2 = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} (1)^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot (N+1)$$

$$= \lim_{N\to\infty} \frac{N(1+1/N)}{N(2+1/N)} = (1/2)$$

Since Energy is infinite
while power is finite so this will be power signal

Q. The energy of signal

$$x[n] = (-0.5)^n u(n)$$

will be

a) 3/4 b) ∞ c) 4/3 (✓) d) 1/2

Sol.ⁿ

$$E = \sum_{n=0}^{\infty} (-0.5)^{2n} = \sum_{n=0}^{\infty} (0.5)^{2n}$$

$$= \sum_{n=0}^{\infty} (0.25)^n$$

$$= 1 + 0.25 + (0.25)^2 + \cdots \infty$$

$$= \frac{1}{1-0.25} = \frac{1}{0.75} = (4/3)$$

Q. Calculate Energy & power of signal.

$$x(n) = (1/2)^n u[n]$$

$$E = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u[n]^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = 1 + \frac{1}{4} + \frac{1}{16} + \cdots$$

$$= \frac{1}{1-\frac{1}{4}} = 4/3$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} [x(n)]^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n = 0$$

Q. $$x[n] = \cos(n\pi/4) = \frac{e^{jn\pi/4} + e^{-jn\pi/4}}{2}$$

$$E = \sum_{n=-\infty}^{\infty} \cos^2(n\pi/4)\, u(n)^2$$

$$= \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \times \sum_{n=-N}^{N} \frac{|e^{jn\pi/4}|^2}{4} + \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N}$$

$$= 1/2$$

Calculation of time period for discrete time system →

$$x(n) = e^{j\omega_0 n} \qquad x(t) = e^{j\omega_0 t}$$

$$x[n+N] = e^{j\omega_0(n+N)}$$

Let N is time period for discrete time system

$$e^{j\omega_0 N} = 1$$

$$e^{j\omega_0 N} = e^{j2\pi}$$

$x[n] = x[n+N]$ by property.

$$\omega_0 = \frac{2\pi}{N} \quad \therefore \boxed{N = \frac{2\pi}{\omega_0}}$$

Difference b/w $x(t) = e^{j\omega_0 t} \rightarrow T_0 = \frac{2\pi}{\omega_0} \rightarrow e^{j\omega_0 t}$ is always Periodic for any value of T_0

$x[n] = e^{j\omega_0 n} \rightarrow N_0 = \frac{2\pi}{\omega_0}$

$T_0 = \frac{2\pi}{\omega_0} \rightarrow e^{j\omega_0 t}$ is always periodic for any value of T_0. This T_0 may be rational or irrational.

$e^{j\omega_0 n}$ is periodic for certain value of N_0.

→ N_0 must be always a rational No.

→ If N_0 is irrational no. then $e^{j\omega_0 n}$ is non-periodic

Ex. $x(t) = e^{3jt} \rightarrow \omega_0 = 3,\ T_0 = \frac{2\pi}{3} \rightarrow$ periodic

$x[n] = e^{3jn} \rightarrow \omega_0 = 3,\ N_0 = \frac{2\pi}{3} \rightarrow$ Non-periodic

Q. $x(t) = \cos\left(\frac{2\pi t}{12}\right)$

$\omega_0 = \pi/6$

$T_0 = 12$

$x[n] = \cos\left(\frac{2\pi n}{12}\right)$

$\omega_0 = \pi/6$

$N_0 = 12$ rational No.

Periodic.

Q. $x(t) = \cos \frac{8\pi t}{31}$ $x[n] = \cos \frac{8\pi n}{31}$

$\omega_0 = 8\pi/31$ $N_0 = (31/4) \approx 31$

$T_0 = 31/4$ $N_0 = 31$ Min. integer value.

Note:- In discrete time domain period N_0 is always integer value. But in continuous time there is no restriction for T_0.

Q. $x[n] = e^{j(3\pi/4)n}$

$\omega_0 = \frac{3\pi}{4}$

$N_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi/4} = 8/3$

$N_0 = 8$

Q. $x[n] = e^{j2\pi n/5}$

$N_0 = 5$

Q. $x[n] = e^{j(3n/4)}$

$\omega_0 = 3/4$

$N_0 = \frac{2\pi}{\omega_0} = \frac{8\pi}{3}$ Not a rational no. therefore Non-periodic.

Q. $x[n] = e^{j(2\pi/3)n} + e^{j(\frac{3\pi}{4})n}$

$\downarrow$ $\downarrow$

$N_0 = 3$ $N_0 = \frac{8}{3}$

$\downarrow$ $\downarrow$

$\frac{3}{1}$ $\frac{8}{1}$ $\rightarrow$ LCM — HCF

$N_0 = 24$

03/10/07

$$E(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt \rightarrow \text{Cont. signal}, \quad P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2$$

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 \rightarrow \text{Discrete signal} \quad P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} [x(n)]^2$$

Q. if $x(n) = u[n]$

What is Energy of signal?

Ans.

$$E = \sum_{n=-\infty}^{\infty} |u(n)|^2$$

$$= \sum_{n=0}^{\infty} (1)^2 = \infty$$

$$u(n) = 1 \quad n > 0$$
$$= 0 \quad n < 0$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} (1)^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot (N+1)$$

$$= \lim_{N\to\infty} \frac{N(1+1/N)}{N(2+1/N)} = (1/2)$$

Since Energy is infinite while power is finite so this will be power signal

Q. The energy of signal

$$x[n] = (-0.5)^n u(n)$$

will be

a) 3/4 b) ∞ c) 4/3 (✓) d) 1/2

Soln

$$E = \sum_{n=0}^{\infty} (-0.5)^{2n} = \sum_{n=0}^{\infty} (0.5)^{2n}$$

$$= \sum_{n=0}^{\infty} (0.25)^n$$