5. $y[n] = x[-n^2] \rightarrow$ causal

$$y[-3] = x[-9]$$

Time- Invariant or Time variant system →

If I/p is shifted by T_0 & o/p is also shifted by same amount then system is called as time-invariant system. otherwise are called as time-variant systems.

Q. $y(t) = \sin x(t) \rightarrow$ check Time variant / In-variant.

Ans.

Step-I $x_1(t) = x(t - t_0)$

$y_1(t) = \sin x(t - t_0)$ — (1)

Step 2:- $y_2(t) = y(t - t_0)$

$= \sin x(t - t_0)$ — (2)

$\therefore y_1(t) = y_2(t)$

So, it is called as Time-invariant System.

Q. $y(t) = t \sin x(t) \rightarrow$ Check Time Variant / In-variant.

S-I $x_1(t) = x(t - t_0)$

$y_1(t) = t \sin x(t - t_0)$ — (1)

S-II $y_2(t) = y(t - t_0)$

$= (t - t_0) \sin x(t - t_0)$ — (2)

$\therefore y_1(t) \neq y_2(t)$

→ So, it is called as Time-variant System.

Q. $y[n] = n\,x[n]$

$y_1[n] = n\,x[n-n_0]$ — (1)

$y_2[n] = (n-n_0)\,x[n-n_0]$ — (2)

$y_1[n] \neq y_2[n]$

↳ Time-variant

Q. $y[n] = x[-n]$

$\cancel{y_1[n] = x[-n-n_0]}$

$\cancel{y_2[n] = x[-n-n_0]}$

S-I $x_1(n) = x(n-n_0)$

$y_1(n) = x(-n-n_0)$

S-II $y_2[n] = y[n-n_0]$

$= x[\cancel{-n} -(n-n_0))$

$= x(-n+n_0)$

$\therefore y_1(n) \neq y_2(n)$

Time-variant

Q.(i) $y[n] = k\,x[n]$ → Time invariant

Q.(ii) $y[n] = x[kn]$ → Time invariant

Ans I $x_1(n) = x_1(n-n_0)$

$y_1(n) = k\,x_1(n-n_0)$

S-II $y_2[n] = y[n-n_0]$

$= k\,x(n-n_0)$

(ii) S-I $y_1[n] = x[kn-n_0]$ —(1)

S-II $y_2[n] = x(k(n-n_0))]$ —(2)

So, it is Time-variant Except $(k=1)$.

Linear - System :- A System is said to be linear if it follows following (both) condition:-

(1) Superposition principle

2. Homogeneous → If zero i/p is given o/p must be zero.

↓

If $x[n]=0$ then $y[n]$ must be zero.

Q. $y[n] = 2x[n] + 3$

if $x[n]=0 \rightarrow y[n]=3$

It does not follow zero i/p, zero o/p condition

so, it is non-linear System.

[Note:- So, $y[n] = ax[n] + b$ is linear if $b=0$ and Non-linear if $b \neq 0$]

Q. check Linearity of $y(t) = t\,x(t)$

Ans. 1. if $x(t)=0$, $y(t)=0$

2. Let $x_3(t) = ax_1(t) + bx_2(t)$

$x_1(t) = y_1(t)$

$x_2(t) = y_2(t)$

$ax_1(t) + bx_2(t) = ay_1(t) + by_2(t)$

Q. 1. $y[n] = n\,x[n]$ - Linear

2. $y[n] = x[n^2]$ - Linear

3. $y[n] = x^2[n]$ - Non- Linear

4. $y[n] = e^{x[n]}$ - Non- Linear if $x[n]=0$, $y[n]=1$.

5. $y[n] = A x[n] + B$. - Non- Linear

Q. $y(t) = u[x(t)]$

if $x(t) = 0 \Rightarrow y(t) \neq 0$
Non- Linear

$\Rightarrow$ So, ZIZO Condition is not valid

Static & Dynamic condition:—

A System is called Static if it is memoryless no. future no. past only present System other than static are called as Dynamic.

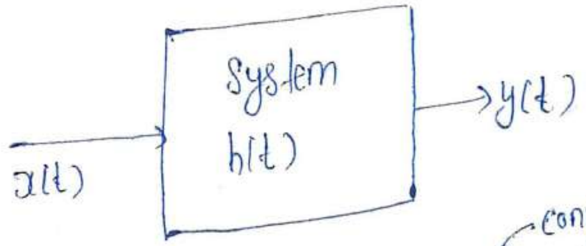
Q. $y(n) = n\,x(n) + b\,x^3(n) \rightarrow$ Static

Q. $y(n) = (n+3)\,x(n) + b\,x^3(n) \longrightarrow$ Static

Q. $y(n) = x(n+3) + b\,x^3(n) \rightarrow$ Dynamic

[:-Note. Always check value of n, as argument of x.]

Convolution



convolution sum symbol

$$y(t) = x(t) \otimes h(t).$$
$$= h(t) \otimes x(t).$$

Q. $x[n] = \alpha^n u[n]$

$h[n] = u[n]$

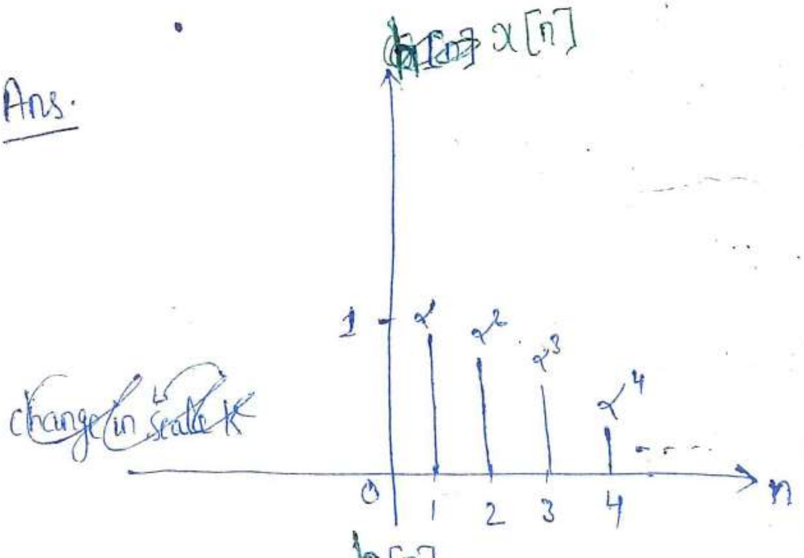
[Note:- $y[n] = x(n) \otimes h(n)$

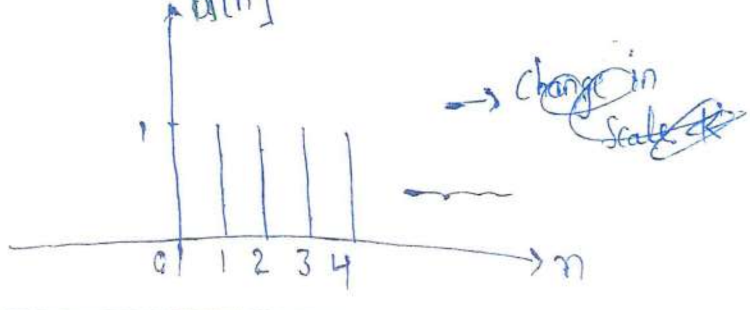
$$= \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

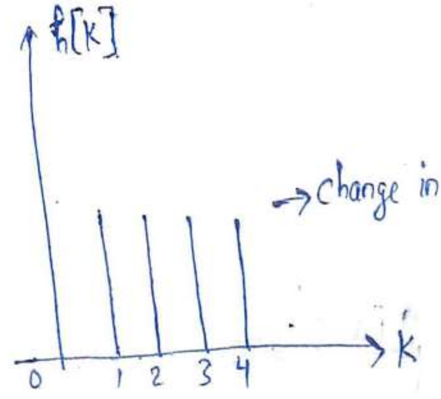
$$y(t) = x(t) \otimes h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$
or
$$= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$
]

Ans.



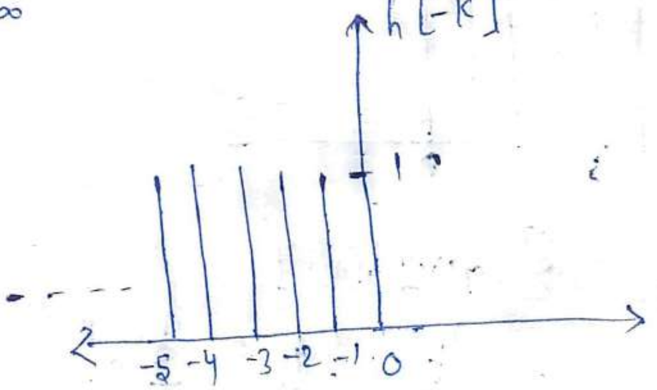


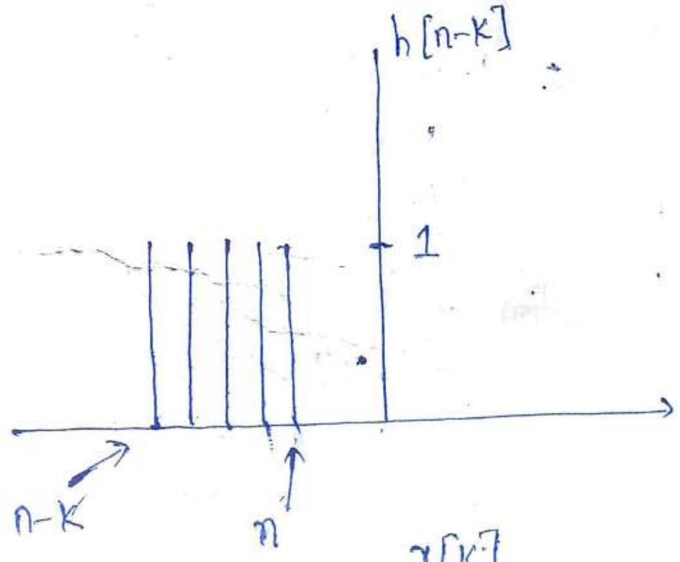
1. $n < 0$
2. $n > 0$

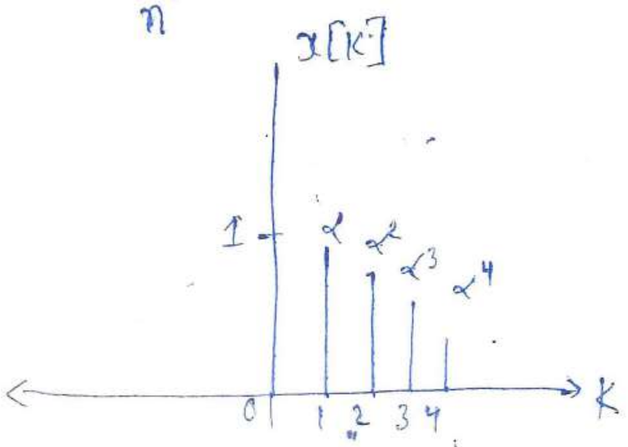


→ change in scale k.

$$\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$





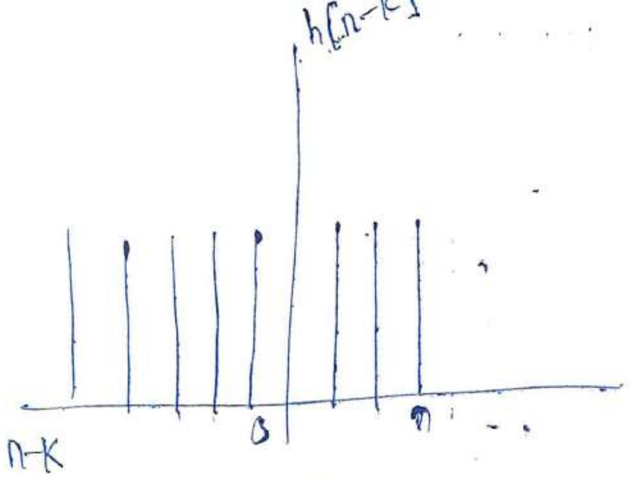


1) $n < 0$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$$= 0$$

2) $n > 0$



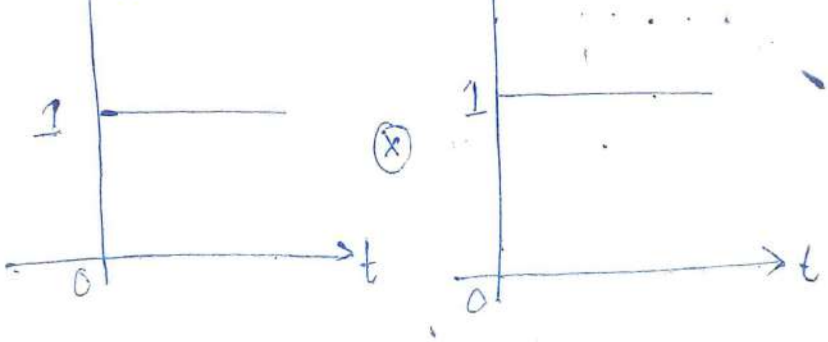
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

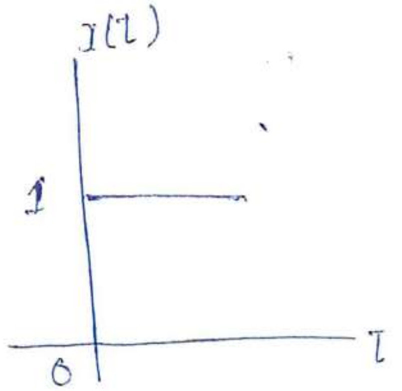
$$= \sum_{k=0}^{n} \alpha^{k} \cdot 1 = \sum_{k=0}^{n} \alpha^{k}$$

$$= 1 + \alpha + \alpha^{2} + \cdots \alpha^{n}$$

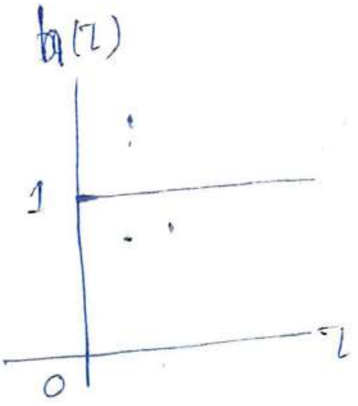
$$= \frac{1 - \alpha^{n+1}}{(1-\alpha)}$$

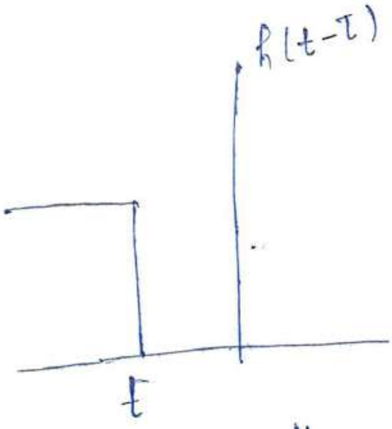
Q. What is convolution of two step function?



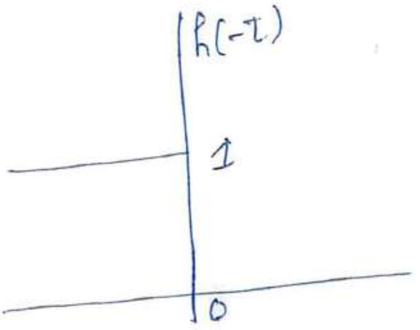


$\otimes$





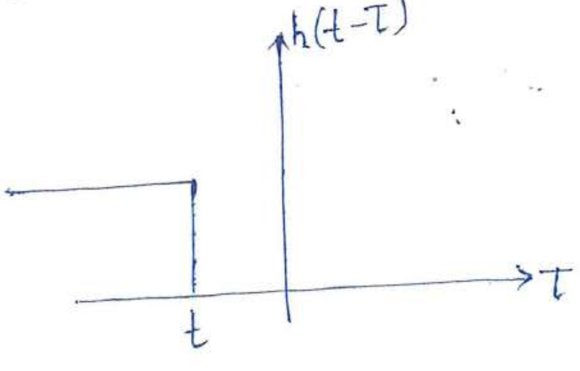
$\Leftarrow$



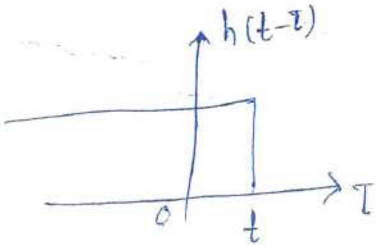
$\Downarrow$

Case I :- $t < 0$

$$y(t) = x(t) \otimes h(t) = 0$$



Case 2:- $t > 0$



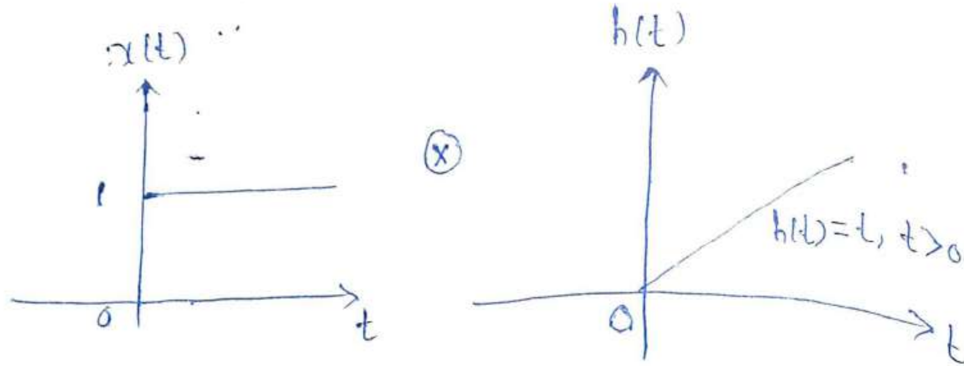
$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

$$= \int_0^t 1 \cdot 1 \, d\tau$$

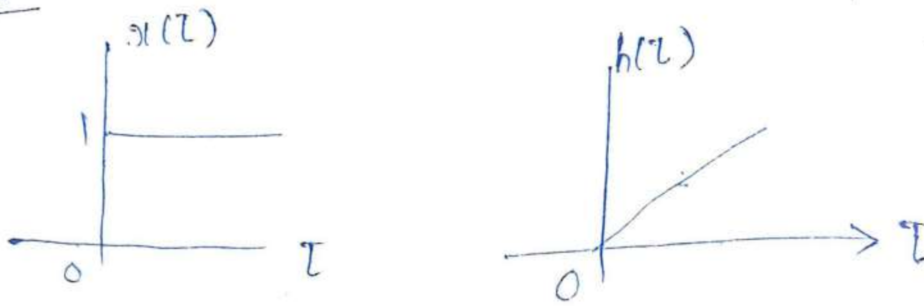
$$= t$$

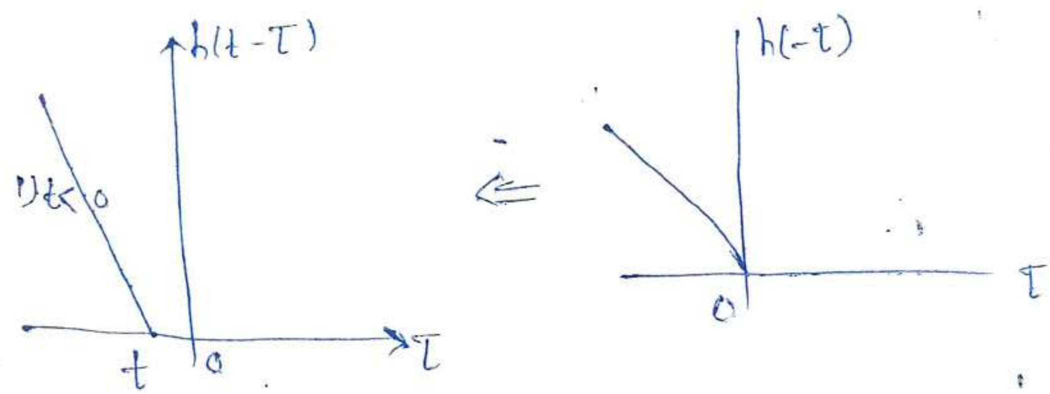
So, convolution of two Step-f^n is Ramp.

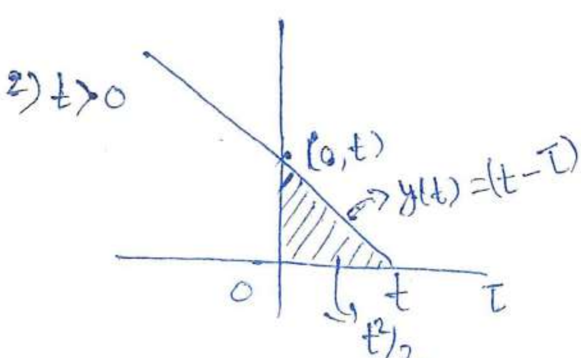
Q.



Ans.







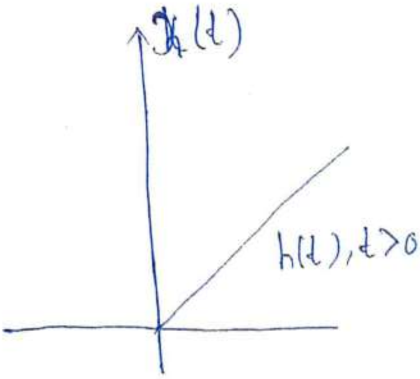
OR

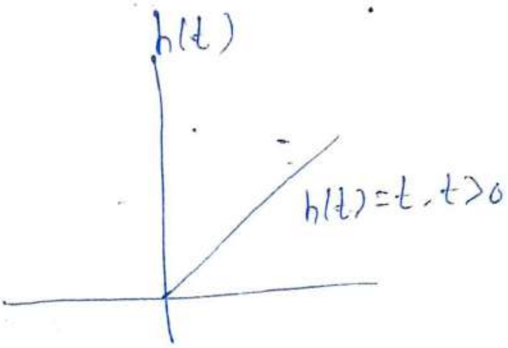
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(\tau)\,d\tau$$

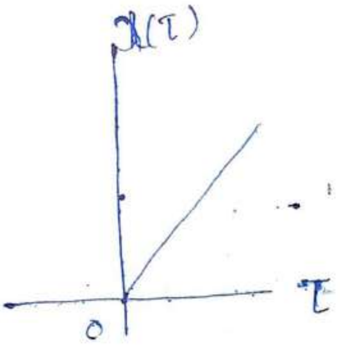
$$= \int_0^t 1\,(t-\tau)\,d\tau$$

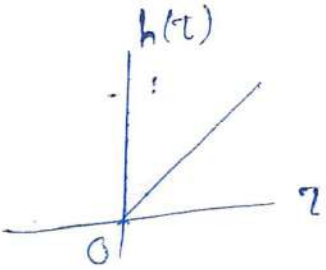
$$= \left| t\tau - \frac{\tau^2}{2} \right|_0^t = t^2 - \frac{t^2}{2} = t^2/2$$

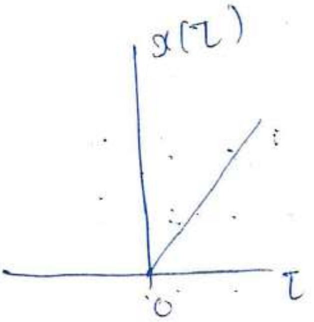
Q. Convolution of two ramp ??

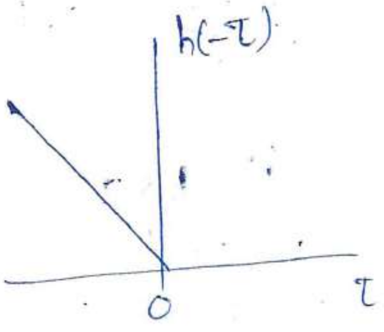


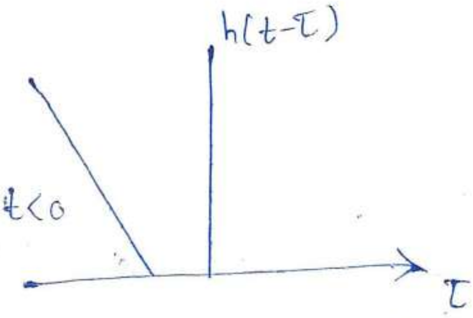


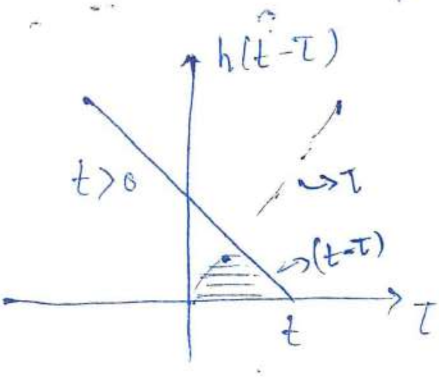












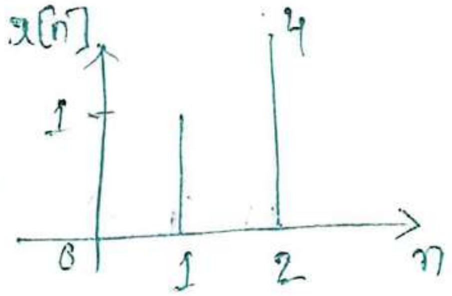
$$y(t) = \int_0^t x(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_0^t \tau(t-\tau)\,d\tau$$

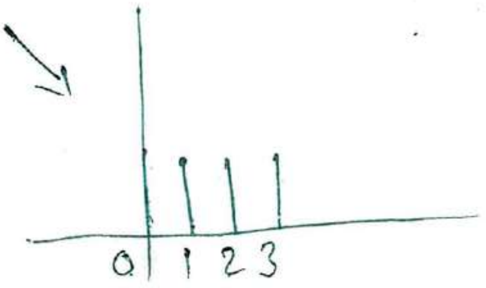
$$= \int_0^t t\tau\,d\tau - \int_0^t \tau^2\,d\tau$$

Discrete convolution

$x[n] = \{1, 2, 4\} \rightarrow$



$h[n] = \{1, 1, 1, 1\}$



$X(Z) = 1 + 2Z^{-1} + 4Z^{-2}$

$h[Z] = 1 + Z^{-1} + Z^{-2} + Z^{-3}$

$Y[Z] = x[Z]\, h[Z]$

$= (1 + 2Z^{-1} + 4Z^{-2})(1 + Z^{-1} + Z^{-2} + Z^{-3})$

$= 1 + Z^{-1} + Z^{-2} + Z^{-3} + 2Z^{-1} + 2Z^{-2} + 2Z^{-3} + 2Z^{-4}$
$\quad + 4Z^{-2} + 4Z^{-3} + 4Z^{-4} + 4Z^{-5}$

$= 1 + 3Z^{-1} + 7Z^{-2} + 7Z^{-3} + 6Z^{-4} + 4Z^{-5}$

$Y[Z] = [1, 3, 7, 7, 6, 4]$

If $x(n)$ has a terms
$y(n)$ " b terms then max. length of sequence $x(n) \circledast y(n) = a + b - 1$.

$x[n] = -4 - j5$

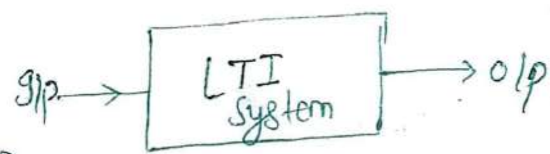
$x^*[n] = -4 + j5$

$$\frac{x[n] - x^*[n]}{2} = (-j5)$$

$$\frac{(1+j2 - (1-j2))}{2} = j2$$

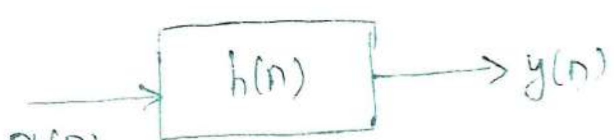
LTI System

1) Linear
2) Time - In-Variant System



1. If i/p is zero then o/p must be zero.
2. If i/p is Linear f'n of certain Equation then o/p will also be a linear f'n of certain Equ.
3. If i/p is shifted by amount t_0, then o/p must be shifted by amount t_0.

1. Discrete - Domain:-



$h[n]$ = Impulse Response or transfer f'n

$$y[n] = x[n] \circledast h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

2. Time domain:-

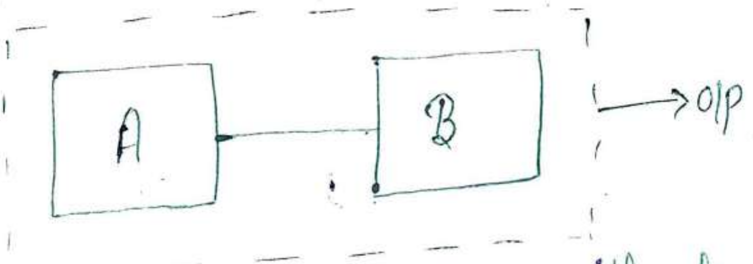
$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t).$$

$$x(t) \otimes h(t) = h(t) \otimes x(t).$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Important Results:-

1 - If A and B are Linear, time invariant &



Causal system then system 'C' will also be Linear, time invariant & causal. But if A & B are non linear, Non causal time variant & unstable then C is not necessary to be Non Linear, time-variant, Non-causal respectively.

Properties of LTI Systems:-

(i) System with or without Memory:-

A System is called Memory less if o/p depend on g/p at the same time only i.e So if $h[n] = 0$, for $n \neq 0$ then System is Memory less

Similarly. if $h(t) = 0$ for $t \neq 0$ then system is memoryless

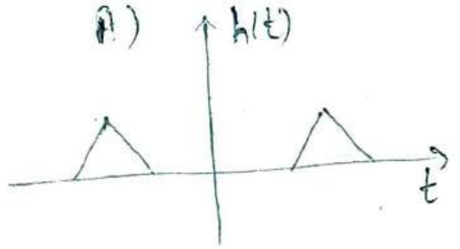
2. Causal System:-

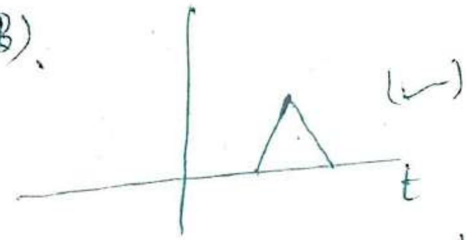
If o/p depends upon present & past only i.e independent of future value then system is called as causal system.

$h(t) = 0$ for $t < 0$ → then System is causal.

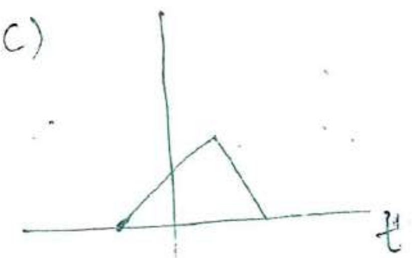
$h[n] = 0$ for $n < 0$ → System is causal.

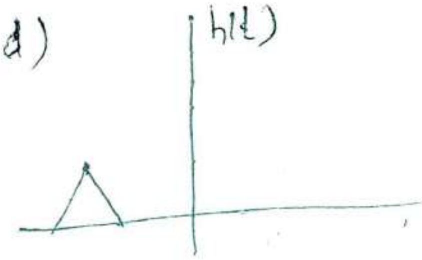
Q. Which one of the following is impulse Response represent a causal system?
GATE-05

A)


B).


$t<0$, $h(t) =$

C)


d)


3

Q. $h[n] = u[n+3] - u[n-2] - 2u[n-7]$

GATE-03

1. D

A) Causal & Memory-less

B) causal & with memory

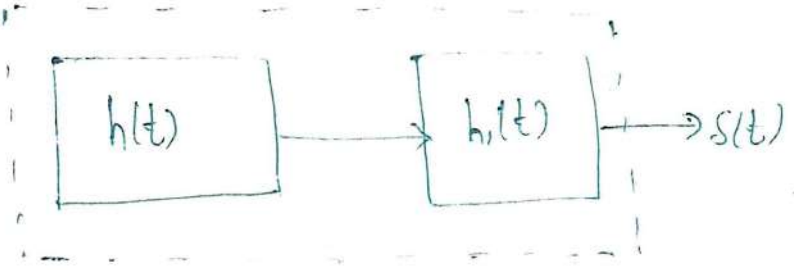
c) NC & Memory less

d) NC with memory (✓)

Invertibility of LTI System:-

- for Invertible System

destinct I/p give distinct o/p i.e one to one mapping exist & there is always Existence of inverse of its function.



$h(t) \otimes h_1(t) = \delta(t)$

Inverse of Impulse response

$\delta(t) \rightarrow$ Impulse f'n.

Q. prove that accumulator is an invertible system.

$$y[n] = \sum_{k=-\infty}^{n} x[k] = y[n-1] + x[n].$$

What is impulse response of accumulator.

A) $\delta[n]$ B) $n\delta[n]$

c) $u[n]$ ✓ d) $nu[n]$

Put $x[n] = \delta[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[n]$$

$$= u[n]$$

$h[n] = u[n]$

Impulse response of accumulator.

for any LTI system, if i/p is given as Impulse, then o/p of LTI system will give its transfer f'n.

Inverse of Accumulator

$$y[n] = y[n-1] + x[n]$$

$$x[n] = y[n] - y[n-1]$$

$\downarrow$ Inverse

$$y[n] = x[n] - x[n-1]$$

$$h_1[n] = \delta[n] - \delta[n-1]$$

→ for invertible system $h[n] \otimes h_1[n]$ must be Equal to $\delta[n]$

$$h[n] \otimes h_1[n]$$
$$= u[n] \otimes [\delta(n) - \delta(n-1)]$$
$$= u(n) \otimes \delta[n] - u(n) \otimes \delta[n-1]$$
$$= u[n] - u[n-1]$$
$$h[n] \otimes h_1[n] = \delta[n]$$

$$\left[x(t) \otimes \delta(t) = x(t) \right.$$
$$\left. x(t) \otimes \delta(t-1) = x(t-1) \right]$$

$$\delta(t) \longrightarrow u(t)$$

$$u(t) = \int_{-\infty}^{\infty} \delta(t) dt = \sum_{k=-\infty}^{\infty} \delta[n-k]$$
$$= u[n].$$

$$\delta(t) = \frac{d}{dt} u[t]$$

Stability of an L.T.I. System:-

$$\int_{-\infty}^{+\infty} |h(t)| dt < \infty$$

$$\sum_{K=-\infty}^{\infty} |h[K]| < \infty.$$

Q. Derive the condition for stability for an LTI System

$$y[n] = \sum_{K=-\infty}^{+\infty} |x[n-K] h[K]|$$

$$|y(n)| \leq \sum_{K=-\infty}^{\infty} |x(n-K)| |h[K]|$$

for checking stability of System

$$|x(n-K)| \leq \infty$$

$$\leq \sum_{K=-\infty}^{\infty} |h(K)|$$

$$\sum_{K=-\infty}^{\infty} |h(K)| \leq \infty$$

10/10/07

Fourier Series →

(i) why we need fourier series?

2 what is F series?

Ans 1. Communication signal involves signal which are complex in nature & is desirable to resolve them in term of sinusoidal form. Here these are resolve in sinusoidal only because if i/p applied sinusoidal to LTI system the o/p will also sinusoidal with same freq. i/p. Except ~~from~~ for transmission delay and gain.

Ans 2. It is a linear combination of elements of a close set of infinite mutually orthogonal function. Orthogonal means one vector has zero component in direction of other i.e. they have nothing common

e.g. Sine and cosine f^n are orthogonal function

3 # Dirichlet conditions – A function x(t) can be exp represented by a complete set of orthogonal function if it follow Dirichlet condition

1. x(t) is a single valued f^n of variable t within interval (t_1, t_2)
2. x(t) has finite no of discontinuities in interval (t_1, t_2)
3. x(t) has finite no of maxima or minim
4. x(t) is absolutely integrable $\int |x(t)| dt < \infty$

$x(t) = \sum_{K=-\infty}^{\infty} C_k e^{jk\omega_0 t}$

This is the waveform for which fourier series is calculated

C_K = fourier series coefficient.

$$x(t) = + C_{-2}e^{-j2\omega_0 t} + C_{-1}e^{-j\omega_0 t} + C_0 + C_1 e^{j\omega_0 t} + C_2 e^{j2\omega_0 t} +$$

C_0 = D.C value or avg. value

C_1 & C_{-1} = first Harmonics

C_2 & C_{-2} = Second Harmonics

Note:- -ve freq. signal are not physical signal but for mathemathical modelling we use -ve frequency.

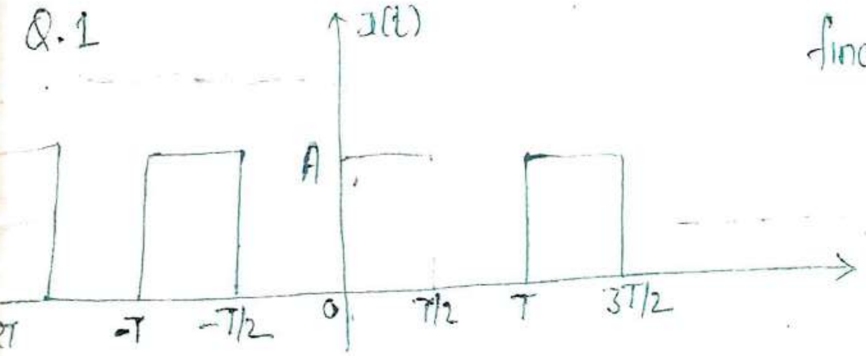
$$\cos\omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$\sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$x(t) = \sum_{K=-\infty}^{\infty} C_K e^{jK\omega_0 t}$

$$C_K = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t}\, dt$$

Q.1 find fourier Series?



Ans.

$$C_K = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_0^{T/2} A \cdot e^{-jk\omega_0 t}\, dt$$

$$\omega_0 = \frac{2\pi}{T}$$

$$= \frac{A}{T \times -jk\omega_0}\left(e^{-jk\omega_0 t}\right)_0^{T/2}$$

$$= \frac{-A}{2\pi jk}\left(e^{-jk \cdot \frac{2\pi}{T} \cdot T/2} - 1\right)$$

$$C_k = \frac{A}{2\pi jk}\left(1 - e^{-jk\pi}\right)$$

$k = \text{odd}$, $\cancel{\&k}\ e^{-j\pi} = \cos(-\pi) + j\sin(-\pi)$

$= 1$

$$C_k = \frac{A}{2\pi jk}\left(1 - e^{-jk\pi}\right)$$

$k = 0 \rightarrow e^{-jk\pi} = 1$

$k = 1 \rightarrow e^{-jk\pi} = -1$

$k = 2 \rightarrow e^{-jk\pi} = 1$

$k = 3 \rightarrow e^{-jk\pi} = -1$

If $k \rightarrow$ odd

$$C_k = \frac{A}{2\pi jk} \times 2$$

$$= \frac{A}{\pi jk}, k = \text{odd}$$

$$C_k = \frac{A}{2\pi jk} \times 0$$

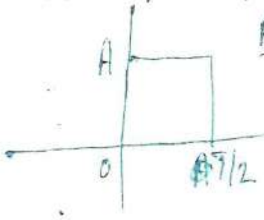
$$= 0, k = \text{even}$$

$$C_k = \frac{A}{\pi j(2m+1)}$$

$\hookrightarrow$ only for odd

$$C_0 = \frac{1}{T}\int_0^{T/2} A \cdot 1\, dt$$

$$= (A/2)$$



$\frac{AT}{2} / T = A/2$

$$x(t) = \frac{A}{2} + \sum_{m=-\infty}^{\infty} \frac{A}{j(2m+1)\pi} e^{+j(2m+1)\omega_0 t}$$

$\hookrightarrow$ Complex fourier series.

fourier Series in sinusoidal form —

$$x(t) = \frac{C_0}{\cancel{2}} + \sum_{K=1}^{\infty} \left(a_K \cos K\omega_0 t + b_K \sin K\omega_0 t\right)$$

$$x(t) = \sum_{K=-\infty}^{\infty} C_K e^{jK\omega_0 t}.$$

$$a_K = 2\,\mathrm{Re}\{C_K\}$$

$$b_K = -2\,\mathrm{Im}\{C_K\}$$

$$C_K = \frac{A}{jK\pi} = \frac{-jA}{K\pi}$$

$a_K = 0$, because has zero real part

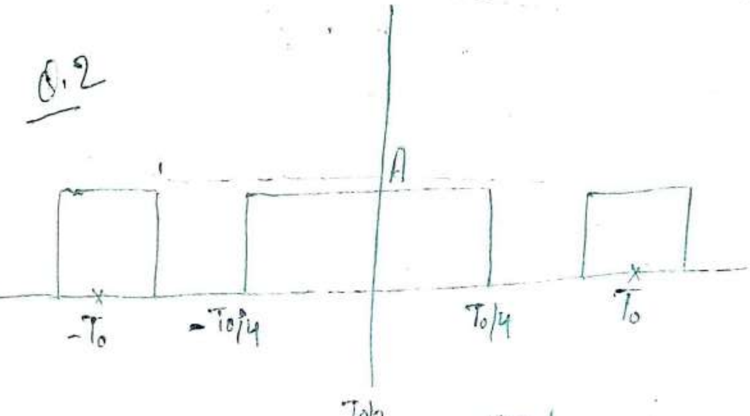
$$b_K = -2 \times \left(\frac{-jA}{K\pi}\right) = \left(\frac{2A}{K\pi}\right). \qquad C_0 = \frac{A}{2}$$

fourier Series in Sinusoidal form —

$$x(t) = \frac{A}{2} + \sum_{K=1}^{\infty} \frac{2A}{K\pi} \sin K\omega_0 t.$$

this will contain sin fn & their odd harmonics.

Q.2



$$C_K = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A e^{-jK\omega_0 t} dt$$

$$= \frac{1}{T_0} \int_{-T_0/4}^{+T_0/4} A e^{-jK\omega_0 t} \cdot dt$$

$$\omega_0 = \left(\frac{2\pi}{T_0}\right)$$

$$= \frac{1 \times A}{T_0(-jk\omega_0)} \times \left(e^{-jk\omega_0 \times T_0/4} - e^{-jk\omega_0 \times -T_0/4}\right)$$

$$= \frac{A}{-jk \cdot 2\pi}\left(e^{-jk\pi/2} - e^{+jk\pi/2}\right)$$

$$= \frac{A}{\pi \cdot k}\left(\frac{e^{jk\pi/2} - e^{-jk\pi/2}}{2j}\right).$$

$$C_k = \frac{A}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

C_k will Exist only for odd value of k.

If $k = 2m+1 \rightarrow C_k = \frac{A}{\pi(2m+1)} \sin \frac{(2m+1)\pi}{2}$

$k = 2m \rightarrow C_k = 0$

$$C_0 = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} A\,dt$$

$$= \left(\frac{A}{2}\right)$$

$$C_0 = \frac{1}{T_0}\int_{-T/2}^{T/2} A\,dt = \left(\frac{A}{2}\right)$$

$$\boxed{x(t) = \frac{A}{2} + \sum_{k=-\infty}^{\infty} \frac{A}{\pi k}\sin\left(\frac{\pi k}{2}\right) e^{+jk\omega_0 t}}$$

$$C_k = \frac{A}{\pi k}\sin\left(\frac{\pi k}{2}\right)$$

$$a_k = \frac{2A}{\pi k}\sin\left(\frac{\pi k}{2}\right)$$

$$b_k = 0$$

$$x(t) = \frac{A}{2} + \sum_{k=-\infty}^{\infty} \frac{2A}{\pi k}\sin\left(\frac{\pi k}{2}\right)\cos k\omega_0 t$$

Sinusoidal fourier series

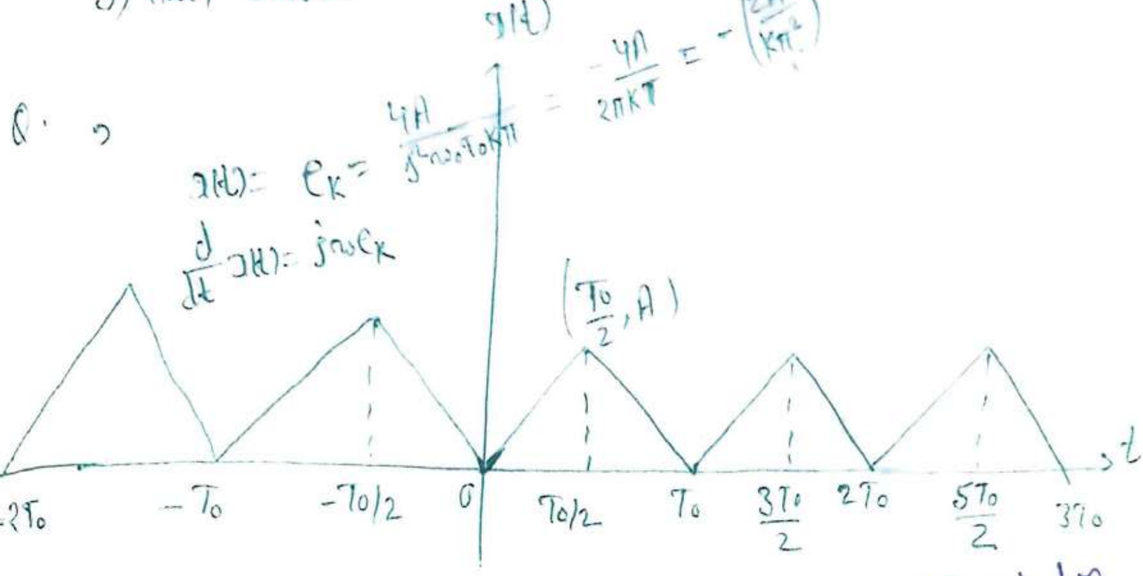
wave - form Symmetry →

1) Even
2) odd
3) Half wave

Q. ?

$x(t)$: $e_K = \frac{4A}{j^2 \omega_0 T_0 K\pi} = \frac{4A}{2\pi K T} = -\left(\frac{2A}{K\pi^2}\right)$

$\frac{d}{dt} x(t) = j\omega e_K$



Ans.

$$x(t) = \begin{cases} \frac{2At}{T_0}, & 0 < t < T_0/2 \\ \frac{2A}{T_0}(T_0 - t) & \frac{T_0}{2} < t < T_0 \end{cases}$$

Differentiation of triangular give square

$d_K = \frac{d}{d\omega} e_K$

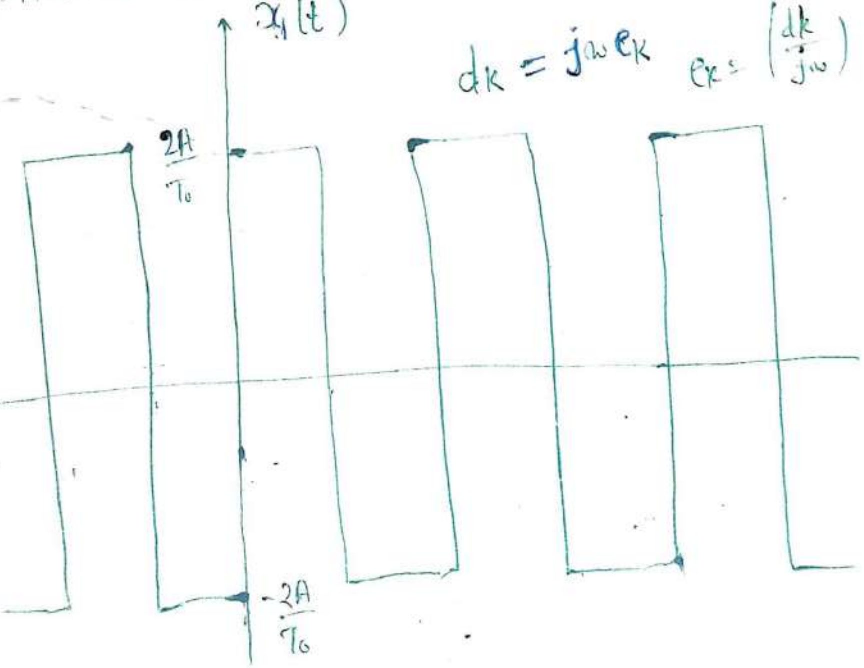
$d_K = j\omega e_K$

ON differentiation →

$d_K = j\omega e_K$ $\quad e_K = \left(\frac{d_K}{j\omega}\right)$

$d_K = \frac{4A}{jT_0 K\pi}$

we have left avg value here So, d_K & e_K value are same.



$\frac{2A}{T_0}$

$-2A/T_0$

OCR...

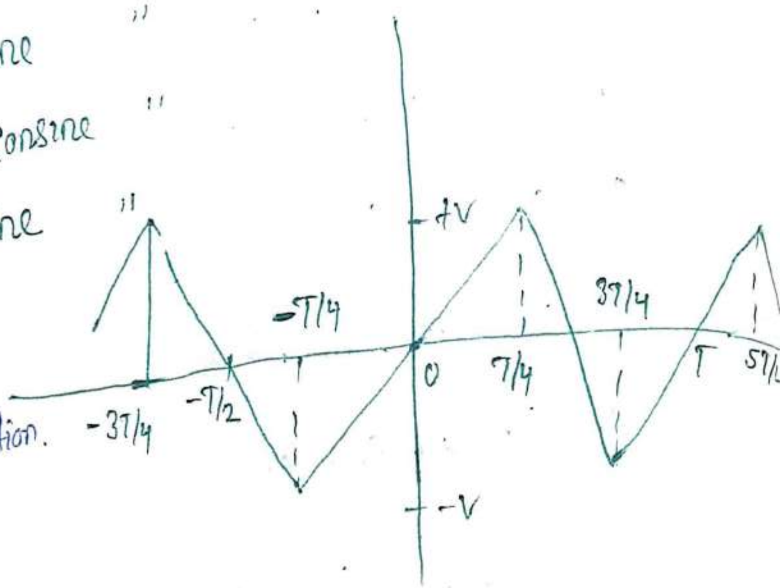
Q. A periodic triangular wave is shown in fig. Its F.series component will consists of

A) All cosine terms

b) All sine "

c) odd consine "

✓ d) odd sine "



Hws & odd function.

Fourier Transform

Q GATE 02 Which of the following cann't have fourier series Expansion

A) $x(t) = 2\cos t + 3\cos 3t$

B) $x(t) = 2\cos\pi t + 7\cos t$ (✓)

c) $x(t) = \cos t + 0.5$

d) $x(t) = 2\cos 1.5\pi t + \sin 3.5\pi t$

Note:- we can write fourier series only for periodic function.

Fourier T/F:-

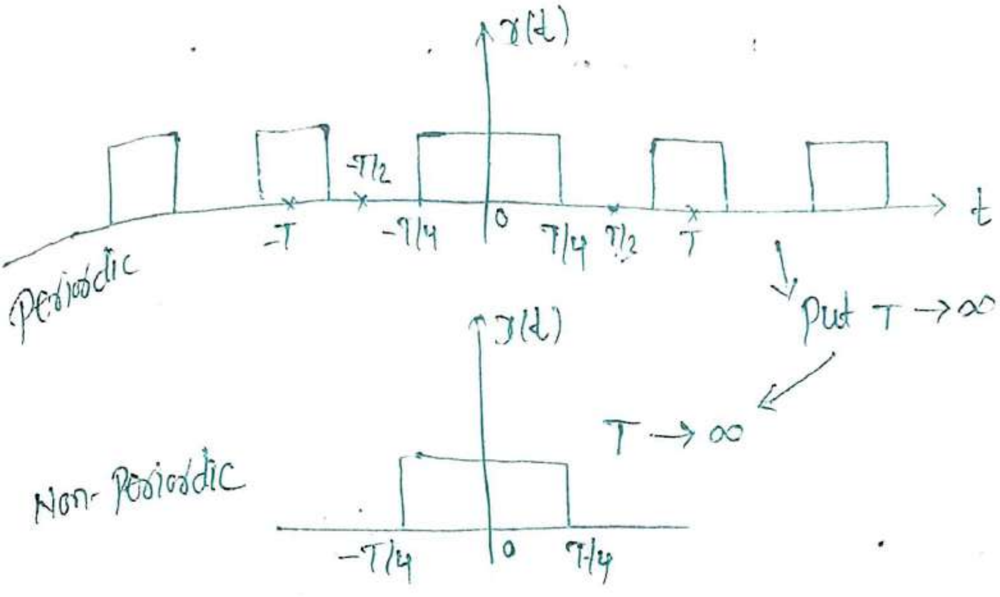
use of impulse ↑

1. Periodic

2. Non Periodic

F.series ✓

1. Periodic



$$\tilde{x}(t) = \sum_{K=-\infty}^{\infty} C_K e^{jK\omega_0 t}$$

$$C_K = \frac{1}{T} \int_{-T/2}^{T/2} \tilde{x}(t) e^{-jK\omega_0 t} dt$$

$$T \cdot C_K = \int_{-T/2}^{+T/2} \tilde{x}(t) e^{-jK\omega_0 t} dt$$

if $T \to \infty$

$$\int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad \boxed{\omega = K\omega_0}$$

So, for here it is clear that fourier transform of $x(t)$ nothing but envelope of $T \cdot C_K$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Fourier transform of $x(t)$.

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \cancel{x(t)}\, X(j\omega) e^{j\omega t} d\omega$$

Condition for F. transform to Exist →

It must for Aperiodic signal it must follows dirichlets condition.

Since for periodic signal

$$\int_{-\infty}^{\infty} |x(t)|\, dt \not< \infty$$

i.e, it is Non absolutely integrable so in this case we will take the help of impulses.

Q. What is F. transform of $x(t) = e^{-at}u(t)$.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega_0 t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= -\frac{1}{a+j\omega} e^{-(a+j\omega)t} \Big|_{0}^{\infty}$$

$$X(j\omega) = \frac{1}{a+j\omega}$$

Q. if $x(t) = e^{at}u(t)$

$$X(j\omega) = \int_{0}^{\infty} e^{(a-j\omega)t}\, dt$$

$$= \int_{0}^{\infty} e^{-(j\omega-a)t}\, dt$$

$$= -\frac{1}{(j\omega-a)} e^{-(j\omega-a)t} \Big|_{0}^{\infty}$$