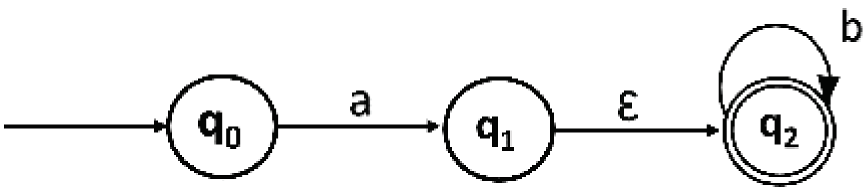
<u>**Eliminating ε Transitions:**</u>

NFA with ε can be converted to NFA without ε, and this NFA without ε can be converted to DFA. To do this, we will use a method, which can remove all the ε transition from given NFA. The method will be:

1. Find out all the ε transitions from each state from Q. That will be called as ε-closure {q1} where qi ∈ Q.
2. Then δ' transitions can be obtained. The δ' transitions mean a ε-closure on δ moves.
3. Repeat Step-2 for each input symbol and each state of given NFA.
4. Using the resultant states, the transition table for equivalent NFA without ε can be built.

<u>**Example:**</u>

Convert the following NFA with ε to NFA without ε.



Solutions: We will first obtain ε-closures of q0, q1 and q2 as follows:

1. ε-closure(q0) = {q0}
2. ε-closure(q1) = {q1, q2}
3. ε-closure(q2) = {q2}

Now the δ' transition on each input symbol is obtained as:

1. δ'(q0, a) = ε-closure(δ(δ^(q0, ε),a))
2. = ε-closure(δ(ε-closure(q0),a))
3. = ε-closure(δ(q0, a))
4. = ε-closure(q1)
5. = {q1, q2}
6.
7. δ'(q0, b) = ε-closure(δ(δ^(q0, ε),b))
8. = ε-closure(δ(ε-closure(q0),b))
9. = ε-closure(δ(q0, b))
10. = Φ

Now the δ' transition on q1 is obtained as:

1. δ'(q1, a) = ε-closure(δ(δ^(q1, ε),a))

2. = ε-closure(δ(ε-closure(q1),a))
3. = ε-closure(δ(q1, q2), a)
4. = ε-closure(δ(q1, a) ∪ δ(q2, a))
5. = ε-closure(Φ ∪ Φ)
6. = Φ
7.
8. δ'(q1, b) = ε-closure(δ(δ^(q1, ε),b))
9. = ε-closure(δ(ε-closure(q1),b))
10. = ε-closure(δ(q1, q2), b)
11. = ε-closure(δ(q1, b) ∪ δ(q2, b))
12. = ε-closure(Φ ∪ q2)
13. = {q2}

The δ' transition on q2 is obtained as:

1. δ'(q2, a) = ε-closure(δ(δ^(q2, ε),a))
2. = ε-closure(δ(ε-closure(q2),a))
3. = ε-closure(δ(q2, a))
4. = ε-closure(Φ)
5. = Φ
6.
7. δ'(q2, b) = ε-closure(δ(δ^(q2, ε),b))
8. = ε-closure(δ(ε-closure(q2),b))
9. = ε-closure(δ(q2, b))
10. = ε-closure(q2)
11. = {q2}

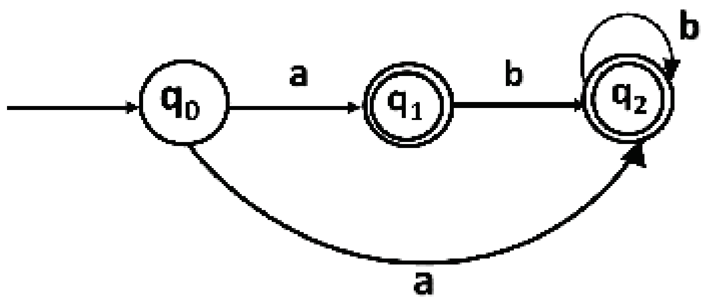
Now we will summarize all the computed δ' transitions:

1. δ'(q0, a) = {q0, q1}
2. δ'(q0, b) = Φ
3. δ'(q1, a) = Φ
4. δ'(q1, b) = {q2}
5. δ'(q2, a) = Φ
6. δ'(q2, b) = {q2}

The transition table can be:

States	**a**	**b**
→q0	{q1, q2}	Φ
*q1	Φ	{q2}
*q2	Φ	{q2}

State q1 and q2 become the final state as ε-closure of q1 and q2 contain the final state q2. The NFA can be shown by the following transition diagram:



Conversion from NFA with ε to DFA:

Non-deterministic finite automata(NFA) is a finite automata where for some cases when a specific input is given to the current state, the machine goes to multiple states or more than 1 states. It can contain ε move. It can be represented as M = { Q, ∑, δ, q0, F}.

Where

1. Q: finite set of states
2. ∑: finite set of the input symbol
3. q0: initial state
4. F: **final** state
5. δ: Transition function

NFA with ∈ move: If any FA contains ε transaction or move, the finite automata is called NFA with ∈ move.

ε-closure: ε-closure for a given state A means a set of states which can be reached from the state A with only ε(null) move including the state A itself.

Steps for converting NFA with ε to DFA:

Step 1: We will take the ε-closure for the starting state of NFA as a starting state of DFA.

Step 2: Find the states for each input symbol that can be traversed from the present. That means the union of transition value and their closures for each state of NFA present in the current state of DFA.

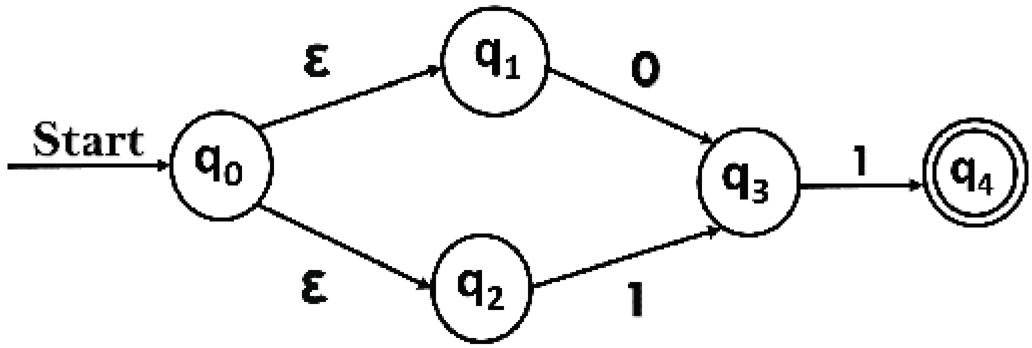
Step 3: If we found a new state, take it as current state and repeat step 2.

Step 4: Repeat Step 2 and Step 3 until there is no new state present in the transition table of DFA.

Step 5: Mark the states of DFA as a final state which contains the final state of NFA.

Example 1:

Convert the NFA with ε into its equivalent DFA.



Solution:

Let us obtain ε-closure of each state.

1. ε-closure {q0} = {q0, q1, q2}
2. ε-closure {q1} = {q1}
3. ε-closure {q2} = {q2}
4. ε-closure {q3} = {q3}
5. ε-closure {q4} = {q4}

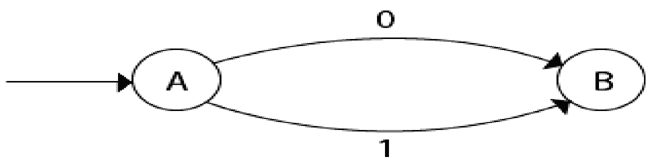
Now, let ε-closure {q0} = {q0, q1, q2} be state A.

Hence

δ'(A, 0) = ε-closure {δ ((q0, q1, q2), 0) }
= ε-closure {δ (q0, 0) ∪ δ (q1, 0) ∪ δ (q2, 0)}
= ε-closure {q3}
= {q3} **call it as state B**.

δ'(A, 1) = ε-closure {δ ((q0, q1, q2), 1) }
= ε-closure {δ ((q0, 1) ∪ δ (q1, 1) ∪ δ (q2, 1)}
= ε-closure {q3}
= {q3} = B.

The partial DFA will be



Now,

δ'(B, 0) = ε-closure {δ (q3, 0)}

= ϕ

δ'(B, 1) = ε-closure {δ (q3, 1) }

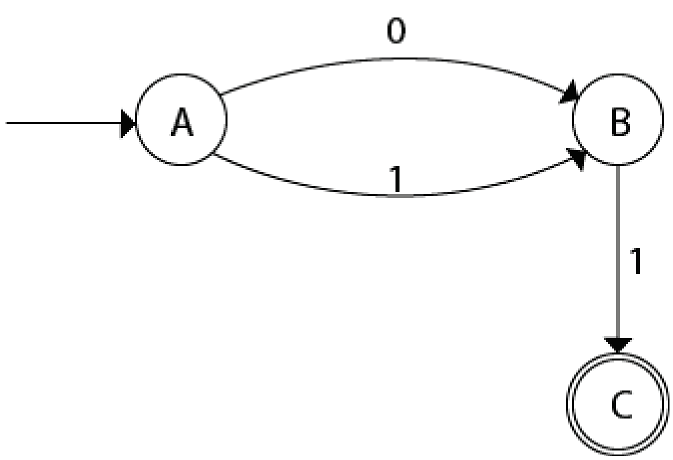
= ε-closure {q4}

= {q4}   **i.e. state C**

For state C:

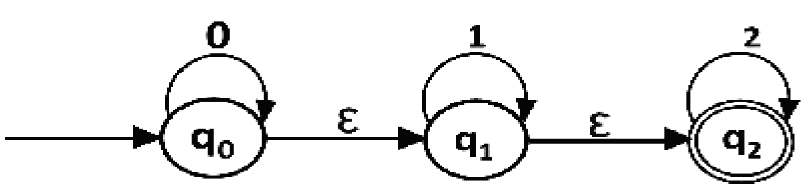
1. δ'(C, 0) = ε-closure {δ(q4, 0) }
2. = ϕ
3. δ'(C, 1) = ε-closure {δ(q4, 1) }
4. = ϕ

The DFA will be,



Example 2:

Convert the given NFA into its equivalent DFA.



Solution: Let us obtain the ε-closure of each state.

1. ε-closure(q0) = {q0, q1, q2}
2. ε-closure(q1) = {q1, q2}
3. ε-closure(q2) = {q2}

Now we will obtain δ' transition. Let ε-closure (q0) = {q0, q1, q2} call it as **state A**.

δ'(A, 0) = ε-closure {δ ((q0, q1, q2), 0)}
= ε-closure {δ (q0, 0) ∪ δ (q1, 0) ∪ δ (q2, 0)}
= ε-closure {q0}
= {q0, q1, q2}

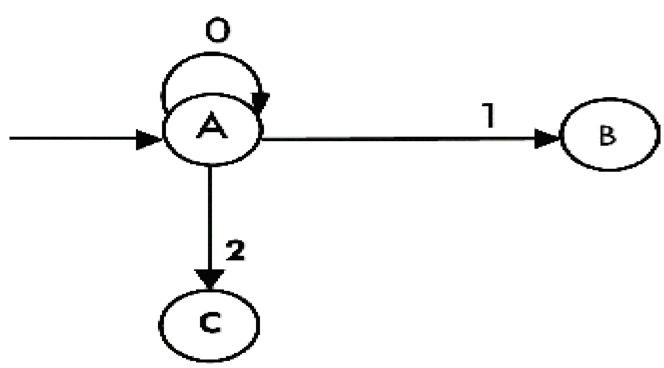
δ'(A, 1) = ε-closure {δ ((q0, q1, q2), 1)}
= ε-closure {δ (q0, 1) ∪ δ (q1, 1) ∪ δ (q2, 1)}
= ε-closure {q1}
= {q1, q2} **call it as state B**

δ'(A, 2) = ε-closure {δ((q0, q1, q2), 2)}
= ε-closure {δ (q0, 2) ∪ δ (q1, 2) ∪ δ (q2, 2)}
= ε-closure {q2}
= {q2} **call it state C**

Thus we have obtained

1. δ'(A, 0) = A
2. δ'(A, 1) = B
3. δ'(A, 2) = C

The partial DFA will be:



Now we will find the transitions on states B and C for each input.

Hence

δ'(B, 0) = ε-closure {δ ((q1, q2), 0)}

= ε-closure {δ (q1, 0) ∪ δ(q2, 0)}

= ε-closure{ϕ}

= ϕ

δ'(B, 1) = ε-closure {δ((q1, q2), 1)}

= ε-closure{δ(q1, 1) ∪ δ(q2, 1)}

= ε-closure{q1}

= {q1, q2} **i.e. state B itself**

δ'(B, 2) = ε-closure{δ((q1, q2), 2)}

= ε-closure{δ(q1, 2) ∪ δ(q2, 2)}

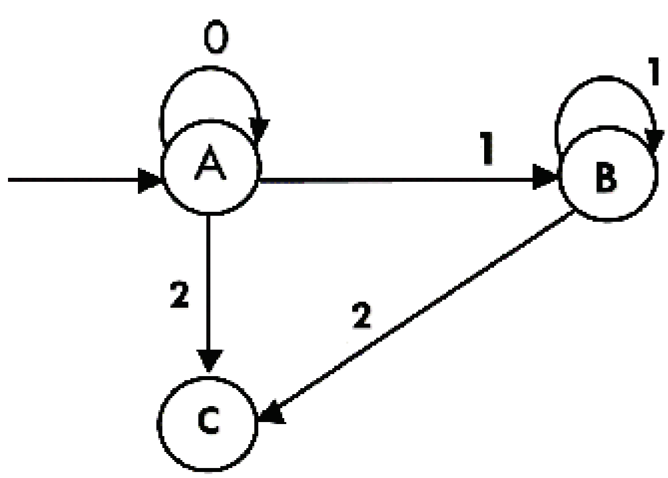
= ε-closure{q2}

= {q2} **i.e. state C itself**

Thus we have obtained

1. δ'(B, 0) = ϕ
2. δ'(B, 1) = B
3. δ'(B, 2) = C

The partial transition diagram will be



Now we will obtain transitions for C:

δ'(C, 0) = ε-closure {δ (q2, 0)}

= ε-closure {ϕ}

= ϕ

δ'(C, 1) = ε-closure {δ (q2, 1)}

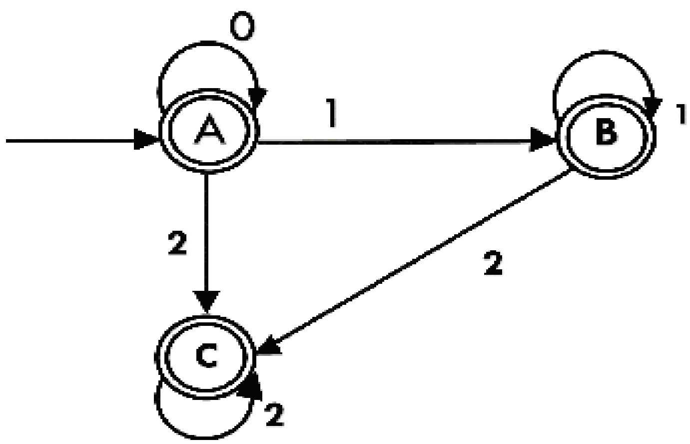
= ε-closure {ϕ}

= ϕ

δ'(C, 2) = ε-closure {δ (q2, 2)}

= {q2}

Hence the DFA is



As A = {q0, q1, q2} in which final state q2 lies hence A is final state. B = {q1, q2} in which the state q2 lies hence B is also final state. C = {q2}, the state q2 lies hence C is also a final state.