Tutorial III, B.Tech. Sem II
Linear Differential Equations of II order with variable coefficients, Simultaneous DE's

1. (i) Solve $x^2y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$, **Ans:** $y = c_1xe^{2x} + c_2x - x^2/2$.
   (ii) Solve $y_2 - \cot x y_1 - (1-\cot x)y = e^x \sin x$, **Ans:** $y = c_1\, e^{-2x}(2\sin x + \cos x) + c_2e^x - \frac{1}{2}e^x \cos x$.
   (iii) Verify that LHS of $(\sin x - x\cos x)y" - x\sin xy' + \sin xy = x$, vanishes when $y = \sin x$ and hence obtain the general solution of the whole equation. **Ans:** $y = c_1\,x + c_2 \sin x + \cos x$.
   (iv) Solve $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2-1)y = -3e^{x^2}\sin 2x$, **Ans:** $y = e^{x^2}(c_1\cos x + c_2\sin x + \sin 2x)$.
   (v) Solve $(1-x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - (1+x^2)y = x$, **Ans:** $y = \frac{1}{1-x^2}(c_1\cos x + c_2\sin x + x)$.

2. (i) Solve $(1+x^2)^2y_2 + 2x(1+x^2)y_1 + 4y = 0$, **Ans:** $(1+x^2)\,y = c_1(1-x^2) + 2c_2x$.
   (ii) Solve $\cos x\, y'' + \sin x\, y_1 - 2\cos^3 x\, y = 2\cos^5 x$, **Ans:** $y = c_1\, e^{\sqrt{2}\sin x} + c_2\, e^{-\sqrt{2}\sin x} + \sin^2 x$.
   (iii) Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\,\frac{dy}{dx} + y = 4\cos\log(1+x)$,
   Ans: $y = c_1\,\cos\log(1+x) + c_2\sin\log(1+x) + 2\log(1+x)\,\sin\log(1+x)$.
   (iv) Solve $y'' + (3\sin x - \cot x)\,y' + 2\sin^2 x\, y = e^{-\cos x}\sin^2 x$, **Ans:** $y = c_1\, e^{\cos x} + c_2\, e^{2\cos x} + \frac{1}{6}\, e^{-\cos x}$.
   (v) Solve $y'' + \frac{1}{x^{\frac{1}{3}}}\, y' + (\frac{1}{4x^{\frac{1}{3}}} - \frac{1}{6x^{\frac{4}{3}}} - \frac{6}{x^2})\, y = 0$, **Ans:** $y = (c_1\, x^3 + c_2\, x^{-2})e^{-\frac{3}{4}x^{2/3}}$.

3. Solve by using method of variation of parameter
   (i) $y_2 + n^2y = \sec nx$, **Ans:** $y = c_1\,\sin x + c_2\,\cos nx + (\frac{x}{n})\,\sin nx + (\frac{1}{n^2})\,\cos nx\,\log\cos nx$.
   (ii) $\frac{d^2y}{dx^2} - 2\,\frac{dy}{dx} = e^x\sin x$, **Ans:** $y = c_1 + c_2e^{2x} - 1/2e^x\sin x$.
   (iii) $x^2\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} - y = x^2\,e^x$, **Ans:** $y = c_1\,x + c_2\frac{1}{x} + e^x - \frac{1}{x}\,e^x$.
   (iv) $(1-x)\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} - y = (1-x)^2$, **Ans:** $y = c_1\,x + c_2e^x - (x^2+x+1)$.
   (v) $\frac{d^2y}{dx^2} - 2\,\frac{dy}{dx} + y = \frac{1}{x^3}e^x$, **Ans:** $y = (c_1 + c_2x)e^x - \frac{1}{2x}\,e^x$.

4. Solve
   (i) $\cos x\frac{d^2y}{dx^2} + \sin x\,\frac{dy}{dx} - 2\cos^3 xy = 2\cos^5 x$, Hint : $Q_1 = -2$ **Ans:** $y = c_1\,e^{\sqrt{2}\sin x} + c_2e^{-\sqrt{2}\sin x + \sin^2 x)}$.
   (ii) $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^2y = 8x^3\sin x^2$, Hint : $Q_1 = -1$ **Ans:** $y = c_1\,e^{x^2} + c_2e^{-x^2} - \sin x^2$.
   (iii) $(1+x^2)\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 8\cos\log(1+x)$,
   Ans: $y = c_1\cos(\log(1+x) + c_2) + 4\log(1+x)\sin(\log(1+x))$.

5. Solve (i) $\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 2y = 3e^t$, $3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^{2t}$
   Ans: $x = \frac{1}{2}e^{2t} - \frac{3}{11}e^t + c_1\,e^{(-6/5)t}$, $y = \frac{15}{22}e^t - 8\,c_1\,e^{(-6/5)t} + c_2\,e^{-t}$.
   (ii) $(\frac{d}{dt}+2)x + 3y = 0$, $3x + (\frac{d}{dt}+2)y = 2e^{3t}$
   Ans: $x = c_1e^t - c_2e^{-5t} - \frac{3}{8}e^{3t}$, $y = -c_1\,e^t + c_2\,e^{-5t} + \frac{5}{8}e^{3t}$.
   (iii) $t^2\,\frac{d^2x}{dt^2} + t\,\frac{dx}{dt} + 2y = 0$, $t^2\,\frac{d^2y}{dt^2} + t\,\frac{dy}{dt} - 2x = 0$
   Ans: $x = t\,(c_1\,\cos\log t + c_2\,\sin\log t) + t^{-1}(c_3\,\cos\log t + c_4\,\sin\log t)$,
   $y = t\,(c_1\,\sin\log t - c_2\,\cos\log t) + t^{-1}(-c_3\,\sin\log t + c_4\,\cos\log t)$.