Tutorial- 1 **B. Tech. Semester IV -2022**

DEPARTMENT OF MATHEMATICS,

Q.1 Form the partial differential equation by eliminating the arbitrary constants from the following

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, (b) $(x-h)^2+(y-k)^2+z^2=c^2$, (c) $z=(x^2+a)(y^2+b)$.

Q.2 Form the partial differential equation by eliminating the arbitrary functions from the following

(a) $z=e^{ny} f(x-y)$, (b) $z=f(x+ay)+g(x-ay)$, (c) $z=f(x+iy)+F(x-iy)$,

(d) $f(x+y+z, x^2+y^2-z^2)=0$.

Q.3 Solve the following PDE's by Lagrange's method

(a) $p\cos(x+y)+q\sin(x+y)=z$, (b) $yzp+xzq=xy$,

(c) $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$,

(d) $(z^2-2yz-y^2)p+(xy+xz)q=xy-xz$, (e) $y^2p-xyq=x(z-2y)$

(f) $p+3q=5z+\tan(y-3x)$, (g) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=xyz$.

Q.4 Obtain the complete solution of the following PDE's by using standard form I, II, III, IV

(a) $p+q=pq$, (b) $x^2p^2+y^2q^2=z^2$, (c) $p^2+q^2=z$, (d) $(p^3+q^3)=27z$,

(e) $p^2+q^2=x+y$, (f) $z=xp+yq+\log pq$.

Q.5 Solve the following PDE's by Charpit's method

(a) $z=pq$, (b) $xp+yq=pq$, (c) $(p^2+q^2)y=qz$ (d) $z^2=pqxy$

(e) $z=xp+yq+p^2+q^2$, (f) $(p+q)(xp+yq)=1$.

Q.6 Obtain the general solution of $(2y^2+z)p+(y+2x)q=4xy-z$. Also, find the particular solution which passes through the straight line $z=1, y=x$.

Q.7 Find the equation of the surface satisfying $t=6x^3y$ and containing the two lines $y=0, z=0,\quad y=1,\ z=1$.

Answer:

Q1. (a) $-z\frac{\partial z}{\partial x}+x\left(\frac{\partial z}{\partial x}\right)^2+zx\frac{\partial^2 z}{\partial x^2}=0$ and $-z\frac{\partial z}{\partial y}+y\left(\frac{\partial z}{\partial y}\right)^2+zy\frac{\partial^2 z}{\partial y^2}=0$,

(b) $z^2(p^2+q^2+1)=c^2$, (c) $pq=4xyz$.

Q2. (a) $p+q=nz$, (b) $\frac{\partial^2 z}{\partial y^2}=a^2\frac{\partial^2 z}{\partial x^2}$, (c) $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$,

(d) $(y+z)p-(x+z)q=x-y$.

Q3. (a) $\phi[z^{\sqrt{2}}\cot\{\frac{1}{2}(x+y)+\frac{\pi}{8}\},\log\{\cos(x+y)+\sin(x+y)\}-x+y]=0$,

(b) $\phi(x^2-y^2,x^2-z^2)=0$, (c) $\phi(x^2+y^2-2z,xyz)=0$,

(d) $\phi(x^2+y^2+z^2,y^2-2yz-z^2)=0$, (e) $\phi(x^2+y^2,yz-y^2)=0$,

(f) $\phi[y-3x,\ e^{-5x}\{5z+\tan(y-3x)\}]=0$, (g) $\phi(\frac{x}{y},\frac{y}{z},xyz-3u)=0$.

Q4. (a) $z=ax+\frac{ay}{(a-1)}+c$, (b) $z=cx^a y^b$, where $b=\sqrt{1-a^2}$,

(c) $4(1+a^2)z=(x+ay+b)^2$, (d) $(1+a^3)z^2=8(x+ay+b)^3$,

(e) $z+b=\frac{2}{3}(x+a)^{3/2}+\frac{2}{3}(y-a)^{3/2}$, (f) $z=ax+by+\log ab$.

Q5. (a) $2\sqrt{z}=ax+\frac{1}{a}y+b$, (b) $az=\frac{(ax+y)^2}{2}+b$, (c) $z^2=a^2y^2+(ax+b)^2$

(d) $z=bx^a y^{1/a}$, (e) $z=ax+by+a^2+b^2$, (f) $\sqrt{1+a}\ z=2\sqrt{x+ay}+b$.

Q6. $\phi(x-y^2+z,x^2-yz)=0$, $z(1-y)+x-y^2+x^2=1$

Q7. $z=x^3y^3+y(1-x^3)$.

Tutorial 2 (PDEs) B. Tech. Sem IV -2022
MATHEMATICS-IV

Q.1 Solve the following PDE's

(a) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$, (b) $\frac{\partial^4 z}{\partial x^4} - 2\frac{\partial^4 z}{\partial x^3 \partial y} + 2\frac{\partial^4 z}{\partial x \partial y^3} - \frac{\partial^4 z}{\partial y^4} = 0$, (c) $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0$,

(d) $(D^2 + DD' + D' - 1)z = 0$, (e) $(D^2 - 2D')z = 0$.

Q.2 Find the general solution of the following PDE's

(a) $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2} = x$, (b) $(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy$,

(c) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \sin ny$, (d) $4r - 4s + t = 16\log(x + 2y)$,

(e) $(D^2 - DD' - 2D'^2)z = (y-1)e^x$, (f) $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$,

(g) $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^y$.

Q.3 Classify the equation:

(a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$,(b) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, (c) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$,

(d) $(1 - x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + (1 - y^2)\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + 3x^2 y\frac{\partial z}{\partial y} - 2z = 0$ ·

Q.4 Using the methods of separation of the variables, solve

(a) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$,(b) $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, where $u(x, 0) = 4e^{-x}$

(c) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, (d) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

Q.5 Solve the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, where $u(0, t) = 0$, $u(l, t) = 0 \;\; t > 0$,

$$u(x, 0) = \begin{cases} A, & when \;\; 0 < x < \frac{l}{2} \\ 0, & when \;\; \frac{l}{2} < x < l \end{cases} .$$

Q.6 Find the temperature distribution $u(x, t)$ in a thin rod of length l, if the initial temperature through the rod is $f(x)$ the ends $x = 0$ and $x = l$, of the rod are insulated.

Q.7 Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, with the boundary conditions $u(x, 0) = 3\sin \pi x$, $u(0, t) = 0$ and $u(1, t) = 0$ where $0 < x < 1, \;\; t > 0$.

Q.8 Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, for $0 < x < \pi, \;\; 0 < y < \pi$, $u(x, 0) = x^2$, $u(x, \pi) = 0$, $u_x(0, y) = u_x(\pi, y) = 0$.

Answer: Q1. (a) $z = \phi_1(y + x) + \phi_2(y - x)$,

(b) $z = \phi_1(y - x) + \phi_2(y + x) + x\phi_3(y + x) + x^2\phi_4(y + x)$,

(c) $z = \phi_1(y + x) + \phi_2(y - x) + \phi_3(y + ix) + \phi_4(y - ix)$, (d) $z = e^{-x}\phi_1(y) + e^{x}\phi_2(y - x)$,

(e) $z = Ae^{h(x + (h/2)y)}$, where A and h are arbitrary constant.

Q2. (a) $z = \phi_1(y + ax) + \phi_2(y - ax) + \frac{x^3}{6}$,

(b) $z = \phi_1(y + 3x) + x\phi_2(y + 3x) + 10x^4 + 6x^3 y$,

(c) $z = \phi_1(y + ix) + \phi_2(y - ix) - \frac{\cos mx \sin ny}{m^2 + n^2}$,

(d) $z = \phi_1(2y + x) + x\phi_2(2y + x) + 2x^2 \log(x + 2y)$, (e) $z = \phi_1(y + 2x) + \phi_2(y - x) + ye^{x}$,

(f) $z = \phi_1(y + x) + x\phi_2(y + x) + e^{x + 2y} + \frac{x^5}{20}$,

(g) $z = e^{x}\phi_1(y) + e^{-x}\phi_2(y + x) + \frac{1}{2}\sin(x + 2y) - xe^{y}$,

Q3. (a) Elliptic, (b) Parabolic, (c) Hyperbolic (d) if $x^2 + y^2 > 1$; Hyperbolic, $x^2 + y^2 = 1$; Parabolic, $x^2 + y^2 < 1$; Elliptic

Q4. (a) $u(x, y) = (Ae^{\sqrt{k}\,x} + Be^{-\sqrt{k}\,x})e^{2ky}$, (b) $u(x, y) = 4e^{-\frac{1}{2}(2x-3y)}$

Q5. $u(x,t) = \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{n\pi}{4}\, e^{-\left(\frac{n\pi c}{l}\right)^2 t} \sin \frac{n\pi x}{l}$

Q6. $u(x,t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n\, e^{-\left(\frac{n\pi c}{l}\right)^2 t} \cos \frac{n\pi x}{l}$, where $A_0 = \frac{2}{l}\int_0^l f(x)dx$,

$A_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$,

Q7. $u(x,t) = 3 \sum_{n=1}^{\infty} e^{-n^2\pi^2 t} \sin n\pi x$

Q8. $u = \frac{\pi}{3}(\pi - y) + 4 \sum_{n=1}^{\infty} \frac{(-1)^n \sinh n(\pi - y)\cos nx}{n^2 \sinh n\pi}$.